MATH089 Project 1 - Population models

Posted: 08/24/21

Due: 09/03/21, 11:55PM

1 Difference equations

1.1 Mathematics of difference equations

1.2 Fibonacci population model

In 1202 Fibonacci introduced a model of population growth based on discrete time reproduction with death or infertility.

1.2.1 Hypotheses

The formal assumptions within the Fibonacci popoulation model are:

1. Count rabbit pairs, denote by F one male and one female;
2. Assume rabbit pairs do not die;
3. Assume each pair reproduces in a constant time interval of one month;
4. Assume one unit of time from birth to fertility;
5. Assume each rabbit pair reproduces exactly one new rabbit pair;
6. Assume all rabits pairs are fertile.

Denote time by $n \in \mathbb{N}$, and let F_n denote the number of pairs at time n.

1.2.2 Mathematical formulation

The Fibonacci model leads to the relation

$$F_n = F_{n-1} + F_{n-2} \, \text{for} \, n \in \mathbb{N}$$

with initial conditions $F_0 = 0$, $F_1 = 1$. The model exhibits exponential growth as shown in Fig. 1

```
∴ function F(n)
    if ((typeof(n)==Int64) && (n>=0))
      if (n<2)
         return n
      end
      return F(n-1)+F(n-2)
    else
      print("Invalid␣argument\n")
    end
  end
```

```
F
```

```
Julia]
```

1.2.3 Direct computation

•

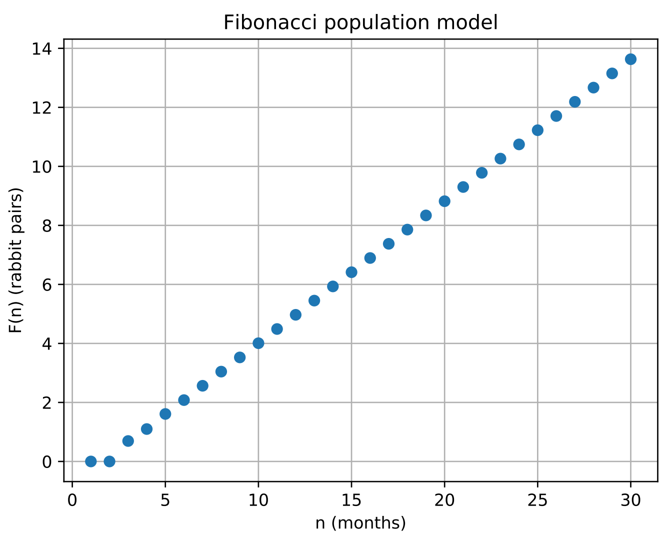

Figure 1. Logarithmic representation of Fibonacci rabbit pair growth.

```
∴ N=30; n=0:N; Fn=F.(n); clf(); plot(n,log.(Fn),"o");
```

```
∴ xlabel("n␣(months)"); ylabel("F(n)␣(rabbit␣pairs)");
```

```
∴ title("Fibonacci␣population␣model"); grid("on");
```

```
∴ savefig(homedir() * "/courses/MATH089/images/Fibonacci.eps")
```

```
∴
```

1.2.4 Comparison of direct computation to analytical solution

1.3 Malthus population model

A different population model is given

$$P_{n+1} = P_n + r\,P_n = (1+r)P_n,\, P_0 = 1.$$

$$P_{n+1} = P_n + r\,P_n = (1+r)P_n,\, P_0 = 1.$$

$$P_{n+1} = P_n + (M - P_{n-1})r\,P_{n-1}$$

$$F_n = F_{n-1} + F_{n-2}\,\text{for}\,n \in \mathbb{N}$$

```
∴ function P(n,r)
    if ((typeof(n)==Int64) && (n>=0) && (r>-1))
      if (n==0)
         return 1
      end
      return (1+r)*P(n-1,r)
    else
      print("Invalid␣argument\n")
    end
  end
```

P

```
∴ P(2,0.1)
```

1.2100000000000002

```
∴
```

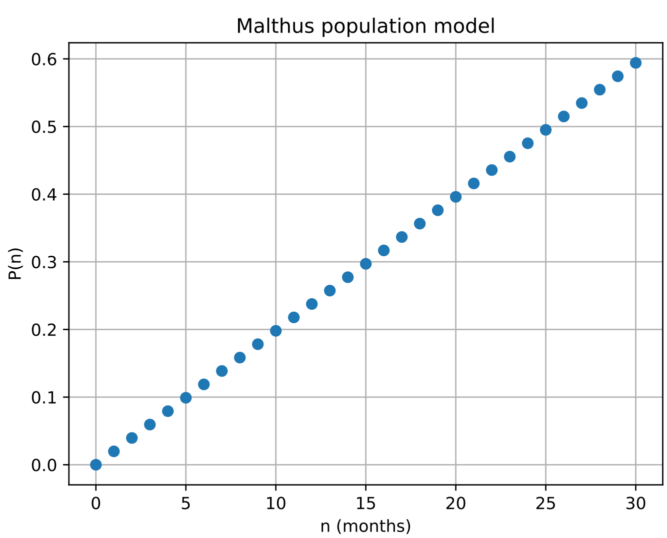

Figure 2.

```
∴ N=30; n=0:N; r=1; Pn=P.(n,r); plot(n,log.(Pn),"o");
∴ xlabel("n␣(months)"); ylabel("P(n)");
∴ title("Malthus␣population␣model"); grid("on");
∴ savefig(homedir() * "/courses/MATH089/images/Malthus.eps")
∴
```

1.4 Logistic population model

2 Systems of difference equations

2.1 Predator-prey models

2.1.1 Hypotheses

-
-
-

2.1.2 Mathematical model

$$\begin{cases} W_{n+1} = W_n + r W_n S_n - a W_n \\ S_{n+1} = S_n - s W_n S_n + b S_n \end{cases}, W_0 = A, S_0 = B$$

2.1.3 Implementation

Julia (1.6.1) session in GNU TeXmacs

```
∴ r=0.01; a=0.02; s=0.005; b=0.1; A=100; B=1000;
```

```
∴ function WS(n)
    global r,s,a,b,A,B
    if ((typeof(n)==Int64) && (n>=0))
      if (n==0)
        return [A B]
      else
        W = WS(n-1)[1] + r*WS(n-1)[1]*WS(n-1)[2] - a*WS(n-1)[1]
        S = WS(n-1)[2] - s*WS(n-1)[1]*WS(n-1)[2] + b*WS(n-1)[2]
        return [W S]
      end
    else
      print("Invalid␣argument")
    end
  end
```

WS

```
∴ WS(3)
```

$$\left[\; -194360.0544 \quad 98038.0068 \;\right] \tag{1}$$

```
∴ x[1]
```

1

```
∴ x[2]
```

2

```
∴
```

2.1.4 Results and discussion

Figure 3.

2.2 Resource-Grazer-Predator models

$$\begin{cases} W_{n+1} = W_n + r W_n S_n - a W_n \\ S_{n+1} = S_n - s W_n S_n + b S_n G_n \\ G_{n+1} = G_n - t S_n G_n + c G_n \end{cases}$$

2.3 Susceptible-Infectious-Recovered disease propagation models

$$\begin{cases} S_{n+1} = S_n - r I_n S_n \\ I_{n+1} = I_n + r I_n S_n - b I_n \\ R_{n+1} = R_n + b I_n \end{cases}$$