

## MATH210: Matrix Computations Syllabus

Single numbers are sufficient to describe some commonly encountered quantities such as an object's weight or a state's population. Many other quantities require more than a single number. Examples include a spatial position, personality traits, or pixel intensities in a medical image. Such quantities are described mathematically through a vector, and groupings of vectors form a matrix. Natural questions are to ask if vector groupings are redundant or sufficient to fully describe some quantity of interest, such as all spatial positions. The answer to such questions is furnished by matrix computations procedures. Furthermore, such computations are the foundation of data science, or the search for regularity in large data sets.

- Vectors and groupings of vectors or matrices
- Changing units of measurement: multiplication of vectors, matrices by a scalar
- Vector addition, matrix addition
- Linear combination: the matrix-vector product
- Multiple linear combinations: the matrix-matrix product
- Redundant vectors: colinearity
- Non-redundant vectors: linear independence
- Efficient non-redundant vectors: orthogonality
- The scalar product, vector norm, unit vectors
- Computing multiple scalar products: the matrix transpose
- Reachable vectors: matrix range or column space
- Unreachable vectors: matrix left null space
- Switching rows and columns: matrix row space, matrix null space
- Minimal vector groupings: bases, dimension
- Relations between row, column, null spaces. Rank-nullity relation.
- Orthogonal bases, the identity matrix
- Change of basis, linear systems
- Triangular ( $LU$ ) factorization to solve linear systems
- Orthogonal factorization ( $QR$ )
- Projection, best reachable approximation, least squares
- Colinear linear combinations  $Ax = \lambda x$ , the eigendecomposition
- Bases for row, column, null spaces, the singular value decomposition (SVD)