L02 - Limit definition

Wednesday, August 17, 2022 4:06 PM

Falling object

- Not a constant velocity

A constant velocity law

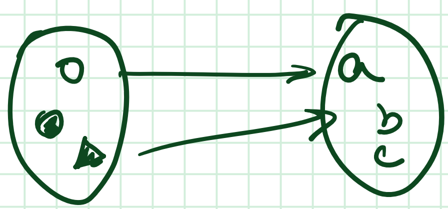
"Distance traversed in time"

Distance denoted by s

Time — " — t

Distance as function of time is denoted

$s(t)$ (Value of s at time t)

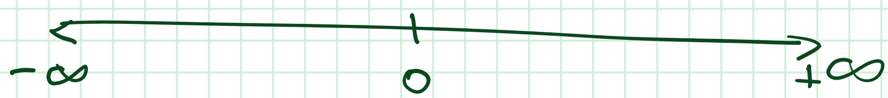


$s : [0, \infty) \to (-\infty, \infty)$

$t \in [0, \infty)$

$s : \mathbb{R}_+ \to \mathbb{R}$

"$\mathbb{R}$" = real axis



If s would have been "constant"

- $s(t) = c$ (some constant)

If s is not constant, but has a constant rate of change

- $s(t) = v \cdot t$

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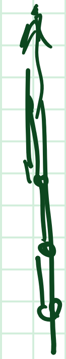
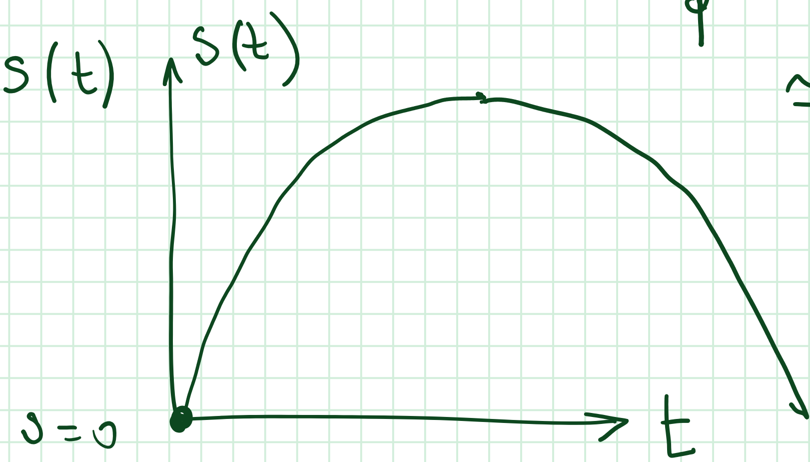
$$s(t) = vt - \frac{gt^2}{2}$$



g acceleration of gravity $\simeq 9.8 \frac{m}{s^2}$

Graph of $s(t)$



Example $\quad v = 5\ m/s$ (initial velocity)

$$g \cong 10\ m/s^2$$

$$s(t) = 5t - 5t^2$$

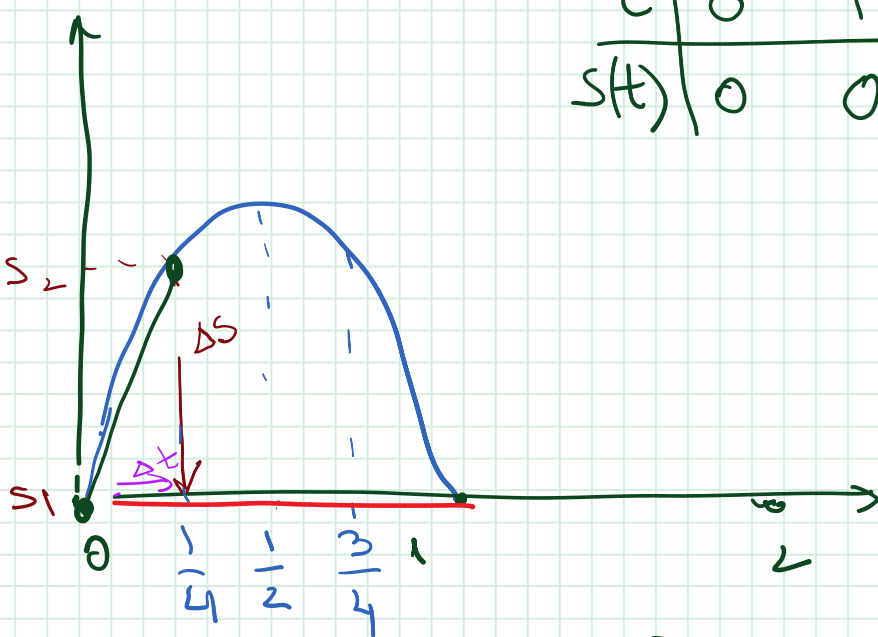
"Apples" = "Apples"

$$\text{meters} = \frac{\text{meters}}{\cancel{\text{second}}}(\cancel{\text{seconds}}) - \frac{\text{meters}}{\cancel{\text{second}^2}}(\cancel{\text{seconds}})^2$$

Rate of change, limits

$s(t) = 5t - 5t^2$

t	0	1	2
$s(t)$	0	0	-10



t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$s(t)$	0	.9375	1.25	.9375	0

Rate of change = velocity = $v(t)$

Average velocity over time interval Δt

Average velocity from $t_1 = 0$ to $t_2 = \frac{1}{4}$

$$\frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{.9375 - 0}{0.25} = 3.75 \ \frac{m}{s}$$

slope of the secant

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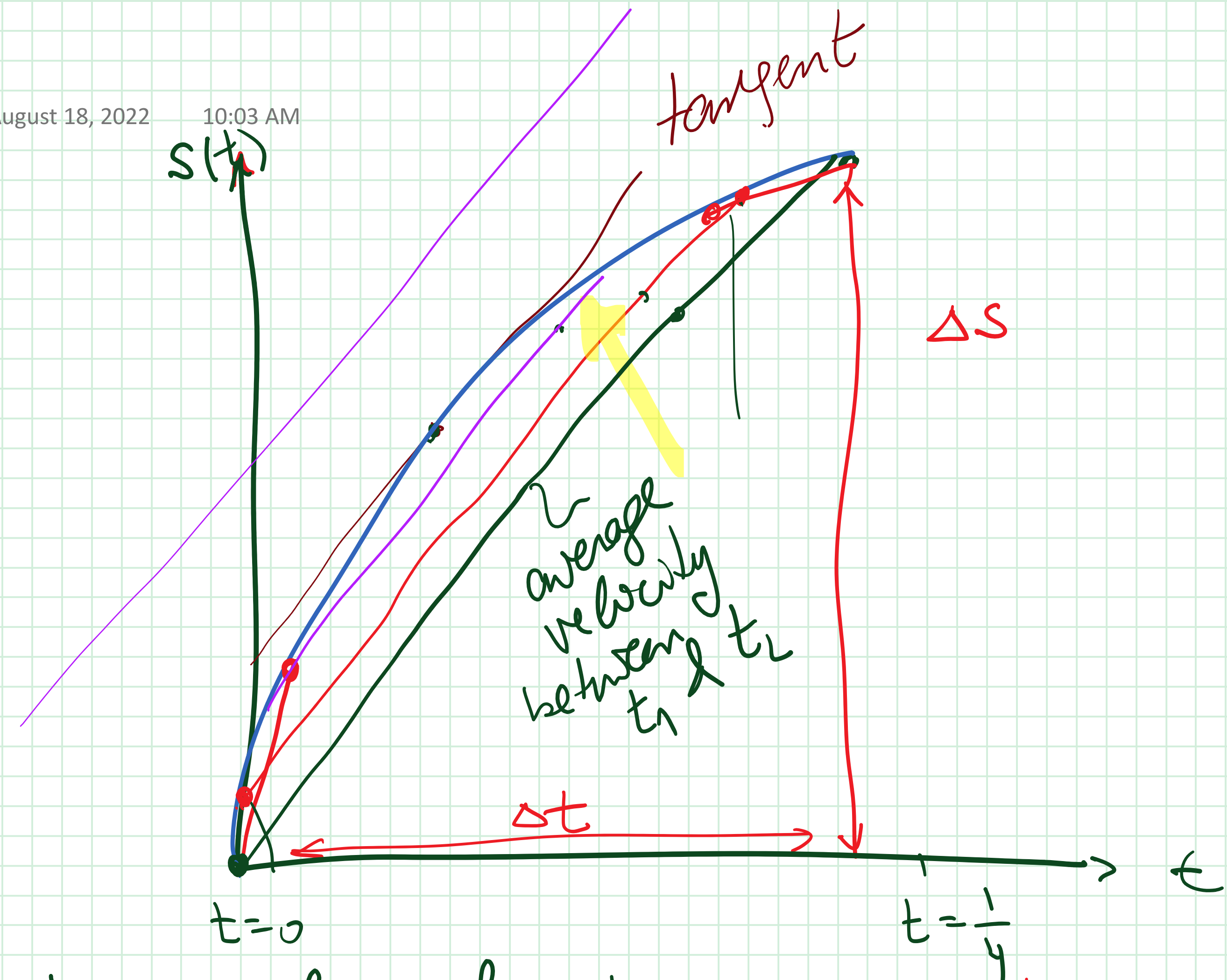

Limit of a function

$$\lim_{x \to a} f(x) = \boxed{L}$$

the limit (if it exists)

value of f at x as it approaches a

x approaches a

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{s(t+\Delta t) \quad s(t)}{\Delta t}$$

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$$f(x)$$

$$x = a + b + c + \sin t + \cos t$$

$$f(a + b + c + \sin t + \cos t)$$

$$\vartheta(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$

Quiz:

$$\cancel{s(t + \Delta t) = s \cdot t + s \cdot \Delta t}$$

$$a(b + c) = a \cdot b + a \cdot c$$

Limit computation

- Numerical approximation

Ex 1) $s(t) = 5t - 5t^2$

$$\lim_{t \to \frac{1}{2}} s(t) = 5 \cdot \frac{1}{2} - 5 \cdot \frac{1}{4} = 5 \cdot \frac{1}{4} = \frac{5}{4}$$

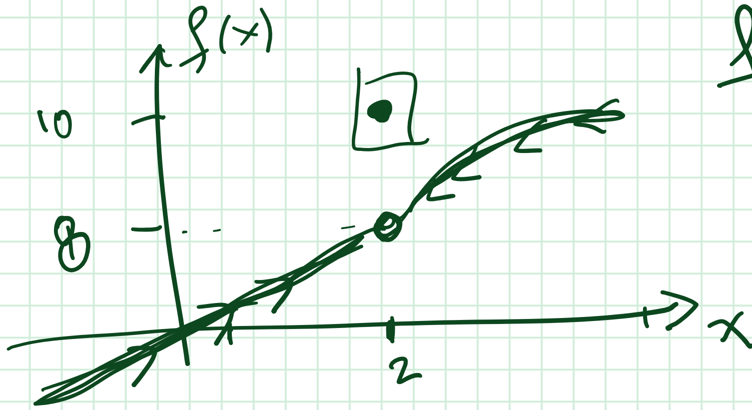
Ex.2) $f(x) = \dfrac{2(x^2-4)}{x-2} = \dfrac{2(x-2)(x+2)}{x-2} = 2(x+2)$

$$\lim_{x \to 2} f(x) = 8$$

$a^2 - b^2 = (a-b)(a+b)$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	7.8	7.98	7.998		8.002	8.02	8.2

Ex 3.



$f(2) = 10$

<u>def</u> f a function $\lim_{x \to a} f(x) = L$ if

$\forall \varepsilon > 0$ $\exists \delta_\varepsilon$ s.t $|x-a| < \delta_\varepsilon$

then $|f(x) - L| < \varepsilon$

Choose your scale

w.r.t. chosen scale choose <u>evaluation</u>

<u>points</u>

Techniques for computing limits

- Numerically (see above)
- Graphically (—"—)
- Analytically

Rules: Suppose $\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to a} g(x) = g(a)$

Then:

1) $\lim_{x \to a} \left(f(x) + g(x) \right) = f(a) + g(a)$

"Limit of a sum" = "sum of the limits" (if they exist)

2) $\lim_{x \to a} \left(f(x)\, g(x) \right) = f(a)\, g(a)$

"Limit of a product" = "Product of limits"

3) $$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}$$

if limits exists & $g(a) \neq 0$.