

2.6.41 Establish intervals of continuity of  $f(x) = \sqrt{5-x}$   
 Establish left/right continuity at endpoints of continuity intervals.

Solution: Square root  $\sqrt{y}$  is defined for positive  $y, y \geq 0$  (State theory)  
 $5-x \geq 0 \Rightarrow 5 \geq x \Leftrightarrow x \leq 5$  (Carry out calculations)

Interval of continuity is  $(-\infty, 5]$  or  
 $f: (-\infty, 5] \rightarrow [0, \infty)$

$\lim_{x \rightarrow 5^-} f(x) = 0$   $f(x)$  is continuous at left (State definitions of left/right continuity)

Since for  $x > 5$   $5-x < 0$ , square root is undefined, function is not continuous at right.

2.6.68.  $f(x) = 2x^3 + x - 2 = 0 \quad x \in (-1, 1)$

a. Use Intermediate Value Theorem (IVT) (Th 2.16) to show  $f(x) = 0$  has a solution for  $x \in (-1, 1)$

IVT states that if  $f: [a, b]$  is continuous and  $f(a) < L < f(b)$  then there exists a number  $c$  such that  $f(c) = L$ . (State theorem)

$f(x) = 2x^3 + x - 2$  is a polynomial, hence continuous  
 $\lim_{x \rightarrow a} f(x) = f(a)$  for all  $a$  (Check conditions)

$$f(-1) = 2 \cdot (-1)^3 - 1 - 2 = -2 - 1 - 2 = -5$$

$$f(1) = 2 \cdot (1)^3 + 1 - 2 = 1$$

$$\text{Since } f(-1) = -5 < 0 < 1 = f(1)$$

IVT implies existence of  $c$  such that  $f(c) = 0$

b. Graph the function



Plot  $2x^3+x-2$

NATURAL LANGUAGE

MATH INPUT

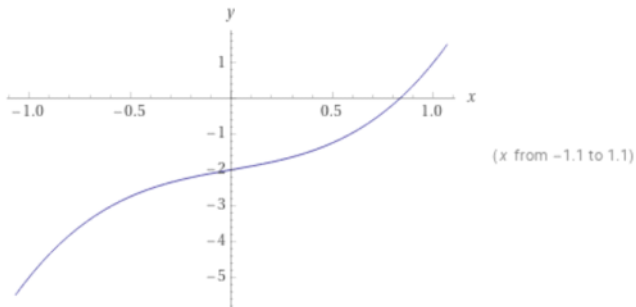
EXTENDED KEYBOARD

Solution is at  
 $x \cong 0.85$

Input interpretation

plot  $2x^3 + x - 2$

Plots



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