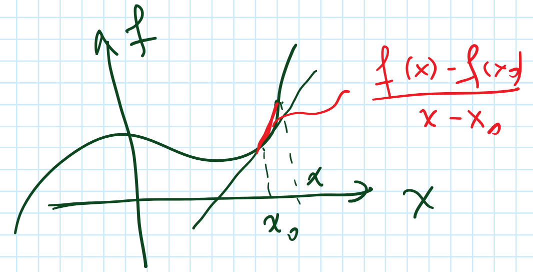
Differentiation rules, derivative function

Thursday, September 8, 2022 9:26 AM



Recall def. of the derivative at a point

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$$f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$$

$f: \mathbb{R} \to \mathbb{R}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$ is the derivative function (if this limit exists)

f	f'
$f(x) = c$ (a constant)	$f'(x) = 0$
$f(x) = x$	$f'(x) = 1$
. . . . . .	
$f(x) = x^n$	$f'(x) = n x^{n-1}$

$n \in \mathbb{N}$

Significance of a derivative

Ex.2

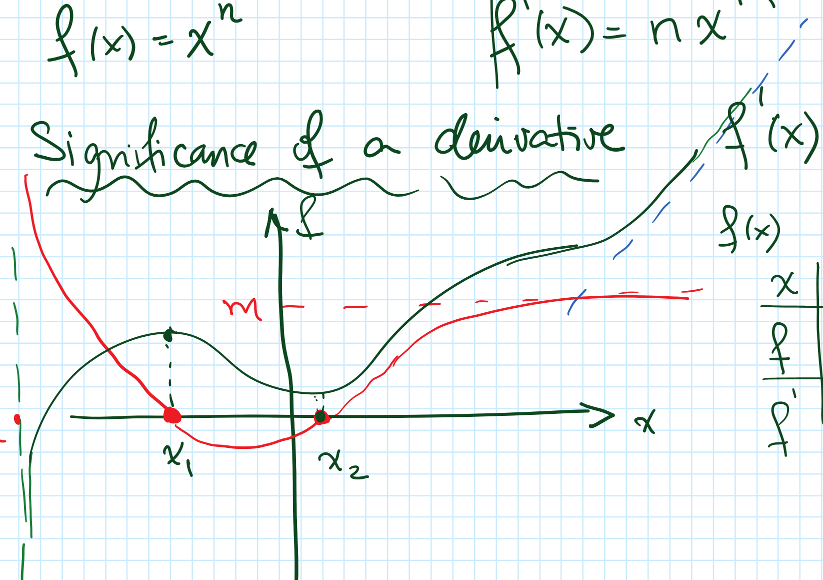
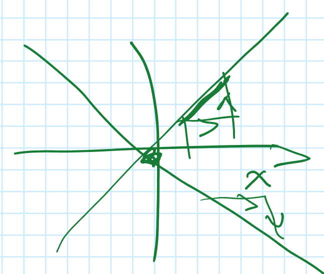
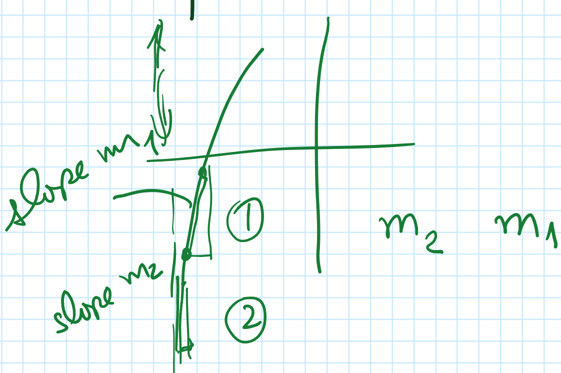


Table (Rolle's)

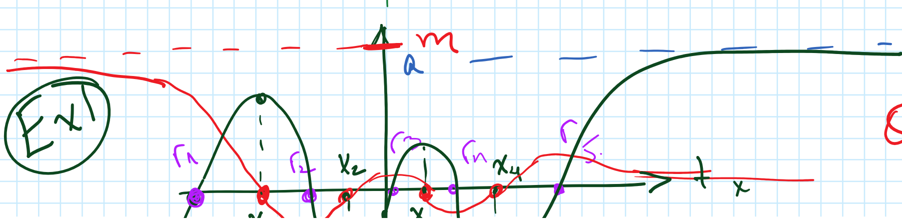
x	a	x_1	0	x_2	∞
f	$-\infty$	+	+	+	∞
f'	∞ +	0	− − − −	0 + +	m

$m > 0$
$m \neq \infty$





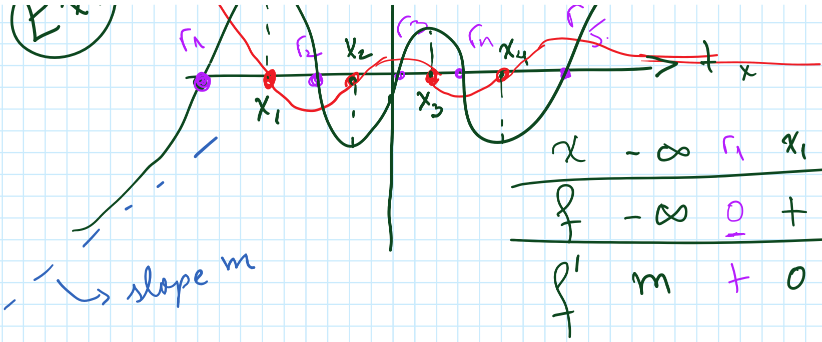
Ex1



r notation because
$r_1, \ldots, r_5$ are called "roots of f"

Qualitative derivative graph
(Rolle table)

Ex.

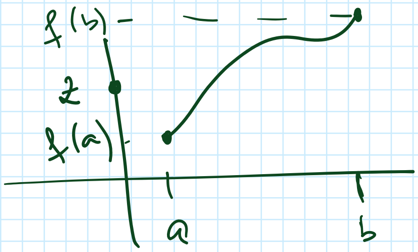


$m > 0$

Qualitative derivative graph
(Rolle table)

x	$-\infty$	r_1	x_1	r_2	x_2	0	r_3	x_3	r_4	x_4	r_5	∞
f	$-\infty$	0	$+$	0	$-$	$-$	0	$+$	0	$-$	0	a
f'	m	$+$	0	$-$	0		$+$	0	$-$	0	$+$	0

Intermediate Value Theorem: If f is continuous on interval $[a,b]$



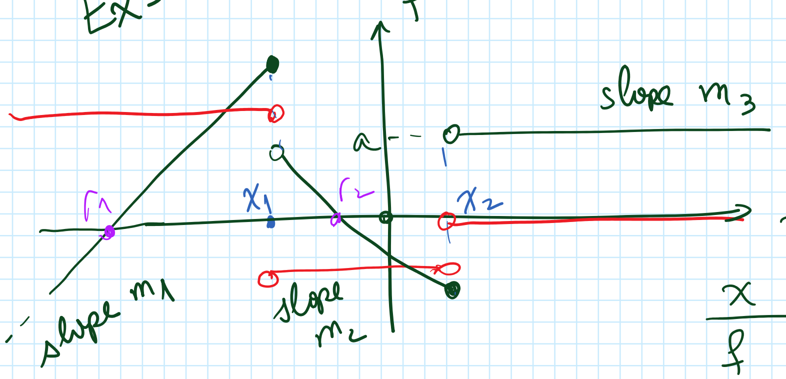
Choose z between $f(a)$ & $f(b)$

$f(a) \le z \le f(b)$ then there must exist c, between a & b

$a \le c \le b$ such that

$$f(c) = z$$

Ex3



x_1, x_2 are points of discontinuity

x	$-\infty$	r_1	x_1	r_2	0	x_2		∞
f	$-\infty$	0		0	$-$			a
f'	m_1	m_1	|	m_2	m_2 m_2	| 0	0	0

Rules of differentiation

$$\lim_{x\to x_0} (f(x) + g(x)) = \lim_{x\to x_0} f(x) + \lim_{x\to x_0} g(x) \quad \text{implies}$$

$$(f+g)' = f' + g' \quad \text{(prime notation for the derivative)}$$

$\frac{d}{dx}$ also denotes differentiation

"Derivative of sum" =

$\frac{d}{dx}$ also denotes differentiation

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

"Derivative of sum" = "Sum of derivatives" *

* = if the individual derivatives exist

$$D(f+g) = Df + Dg$$

(Note that both for D & for $\frac{d}{dx}$

Df

$\frac{d}{dx} f$

we do not mean multiplication

"D" & "$\frac{d}{dx}$" is an "operator"

$$(f+g)' = f' + g'$$

$(cf)'$ c is a constant, f a function

$F = cf$ $F(x) = c f(x)$

$$F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h}$$

$$= \lim_{h \to 0} \left[c \frac{f(x+h) - f(x)}{h} \right] = c f'$$

"Limit of product" = "product of limits" *

Use together

$af + bg$ = "Linear combination"

$$(af+bg)' = (af)' + (bg)' = af' + bg'$$

Ex: $f(x) = 2x^2 + 5x^3$

$$f'(x) = 4x + 15x^2$$

Ex: $$f(x) = -x^3 + x^2 - x + 2$$
$$f'(x) = -3x^2 + 2x - 1$$

Ex: $$p_n(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$
$$p_n'(x) = n a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \dots + a_1$$

- $$\left(\frac{1}{c} f\right)' = \frac{1}{c} f'$$

$0 = 1 - 1$
$0 = 2 - 2$
$0 = c - c$

- $$(f g)' \qquad F(x) = f(x) g(x)$$

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) + 0 - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) \overbrace{- f(x)g(x+h) + f(x)g(x+h)}^{0} - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) - f(x)]g(x+h) + f(x)[g(x+h) - g(x)]}{h}$$

$$= \lim_{h\to 0}\left[\frac{f(x+h) - f(x)}{h} \cdot g(x+h)\right] + \lim_{h\to 0}\left[f(x)\,\frac{g(x+h) - g(x)}{h}\right]$$

$$f'(x)g(x) + f(x)g'(x)$$

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$$(f g)' = f'g + fg'$$ Product differentiation

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$$(fg)' = f'g + fg'$$ Product differentiation Rule

Not $\cancel{(fg)' = f'g'}$

Ex $f(x) = x^2 = x \cdot x$

$f'(x) = 2x$

$f'(x) = (x \cdot x)' = x' \cdot x + x \cdot x'$

$= 1 \cdot x + x \cdot 1 = 2x$ ✓