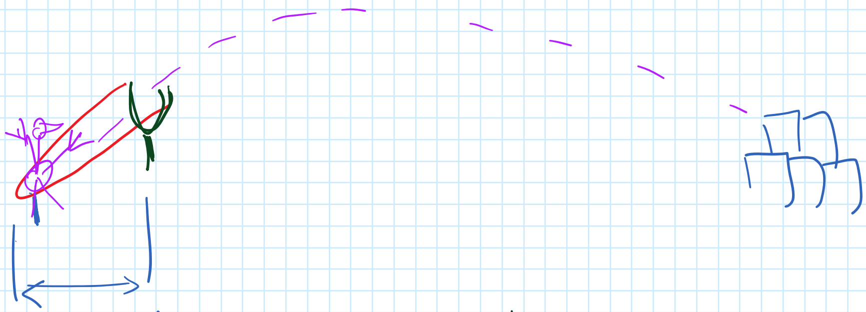
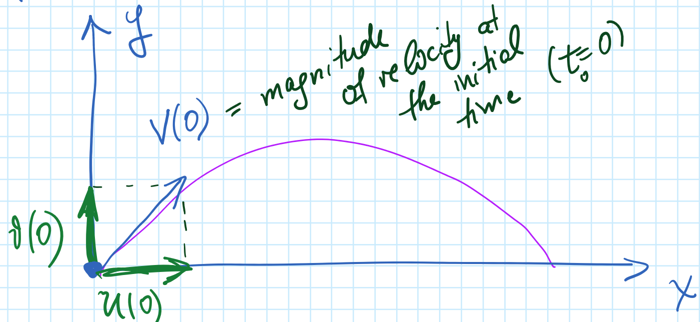
Tuesday, September 20, 2022 10:26 AM





Assume no friction:

$\vec{V}(0)$ = initial velocity vector

$$\vec{V}(0) = \begin{bmatrix} u(0) \\ \vartheta(0) \end{bmatrix}$$

$|\vec{V}(0)|$ = magnitude of initial velocity $= V(0) = \left[u^2(0) + \vartheta^2(0)\right]^{1/2}$

$x(t)$ = position of projectile along horizontal direction

$\frac{dx}{dt}$ = instantaneous velocity $= u_0$

$$x(t) = u(0)\,t \qquad (x(0) = 0 \checkmark)$$

$u_0 \equiv u(0) \qquad \vartheta_0 \equiv \vartheta(0)$ (Notation)

$y(t)$ = position of projectile along vertical direction

$$y(t) = \vartheta_0 t - \frac{gt^2}{2}$$ → effect of gravity

$u(t)$ = velocity in the horizontal direction

$\vartheta(t)$ = velocity in the vertical direction

$$\frac{du}{dt} = 0 \Rightarrow u = \text{constant} = u_0 ; \quad \frac{dx}{dt} = u_0 \Rightarrow$$

$$\frac{du}{dt} = 0 \Rightarrow u = \text{constant} = u_0 ; \quad \frac{dx}{dt} = u_0 \Rightarrow$$

$$x(t) = u_0 t$$
$$x(t) = u_0 t + C$$
$$x(0) = 0 \Rightarrow C = 0$$
$$x(t) = u_0 t$$

$$\frac{d\vartheta}{dt} = -g \Rightarrow \vartheta(t) = -gt + c$$

$$\vartheta(0) = -g \cdot 0 + c = c = \vartheta_0$$

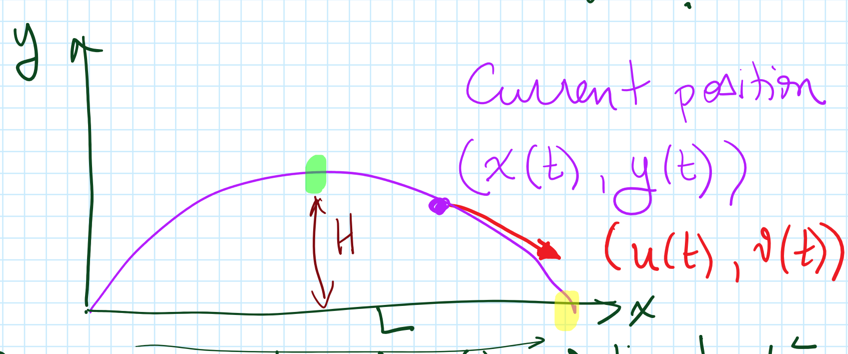
$$\vartheta(t) = \vartheta_0 - gt$$

$$\begin{cases} u(t) = u_0 \\ x(t) = u_0 t \end{cases} \qquad u(t) = \frac{dx}{dt} \qquad \begin{cases} \vartheta(t) = \vartheta_0 - gt \\ y(t) = \vartheta_0 t - \frac{1}{2} g t^2 \end{cases}$$

Motion along horizontal, x-axis

Motion along vertical, y-axis

$$\frac{dt^2}{dt} = 2t \qquad \frac{d}{dt}\left(\underbrace{-\frac{1}{2} g}\, t^2\right) = \left(-\frac{1}{2} g\right) 2t = -gt.$$



Independent variable t

Dependent variables $x(t)$, $y(t)$

$$\begin{cases} u(t) = \frac{dx}{dt} \\ \vartheta(t) = \frac{dy}{dt} \end{cases}$$

$$\begin{cases} x(t) = u_0 t \\ u(t) = u_0 \end{cases} \qquad \begin{cases} y(t) = \vartheta_0 t - \frac{1}{2} g t^2 \\ \vartheta(t) = \vartheta_0 - gt \end{cases}$$

$$H = \cancel{y(z)} = y(t_H)$$

Fancy

$$\vartheta(t_H) = 0 = \vartheta_0 - g t_H \Rightarrow t_H = \frac{\vartheta_0}{g}$$

Unit of measurement check [ ] denotes units of measurement

$$[t_H] = \left[\frac{v_0}{g}\right] = \frac{[v_0]}{[g]}$$

$$\text{sec} = \frac{\frac{m}{sec}}{\frac{m}{sec^2}} \checkmark$$

$$t_H = \frac{v_0}{g} \qquad H = y(t_H) = v_0 t_H - \frac{1}{2} g t_H^2$$

$$= \frac{v_0^2}{g} - \frac{1}{2} g \frac{v_0^2}{g^2}$$

$$= \frac{v_0^2}{g}\left(\frac{1}{2}\right) = \frac{v_0^2}{2g}$$

$$L = x(t_L) \qquad y(t_L) = 0$$

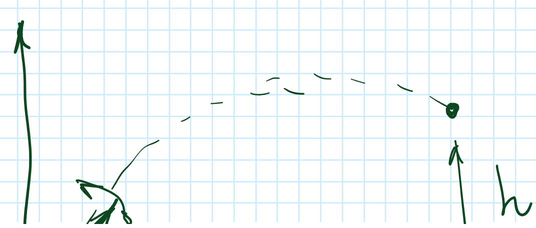
$$y(t_L) = v_0 t_L - \frac{1}{2} g t_L^2 = 0$$

$$t_L \underbrace{\left(v_0 - \frac{1}{2} g t_L\right)}_{\text{verifies units of measurement}} = 0 \rightarrow \cancel{t_L = 0 \text{ (initial)}}$$

$$\rightarrow t_L = \frac{2v_0}{g}$$

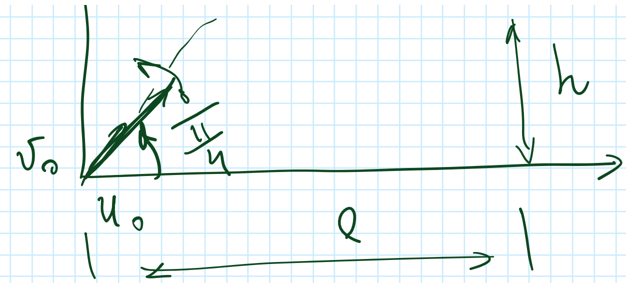
$$L = x(t_L) = u_0 t_L = u_0 \frac{2v_0}{g}.$$

Angry birds



$$u_0 = v_0$$

$$\{ y(t_h) = h$$



$$\begin{cases} y(t_h) = h \\ x(t_h) = \ell \end{cases}$$

$$\begin{cases} x(t) = u_0 t \\ u(t) = u_0 \end{cases} \qquad \begin{cases} y(t) = v_0 t - \frac{1}{2} g t^2 \\ v(t) = v_0 - g t \end{cases}$$

$$x(t_h) = \ell \qquad y(t_h) = h$$

$$u_0 t_h = \ell \qquad v_0 t_h - \frac{1}{2} g t_h^2 = h$$

$$t_h = \frac{\ell}{u_0} \qquad u_0 = v_0$$

$$t_h = \frac{\ell}{v_0} \Rightarrow$$

$$v_0 \left(\frac{\ell}{v_0}\right) - \frac{1}{2} g \left(\frac{\ell}{v_0}\right)^2 = h$$

$$\ell - \frac{1}{2} \frac{g \ell^2}{v_0^2} = h \Rightarrow$$

$$\ell - h = \frac{1}{2} \frac{g \ell^2}{v_0^2} \Rightarrow v_0 = \sqrt{\frac{2(\ell - h)}{g \ell^2}}$$

Typical application of derivatives:

1) Specification of rates of change

2) Place conditions

3) Do the algebra