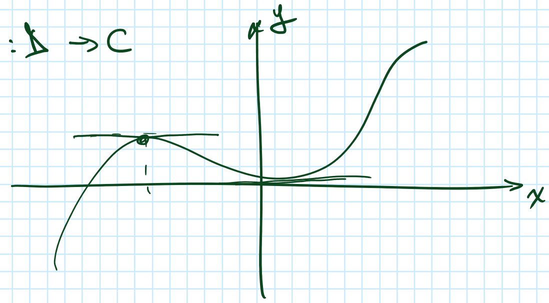
4. Applications of Derivatives

Tuesday, October 11, 2022 9:56 AM

4.1 Extrema

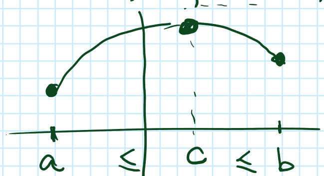
Extrema: points at which a function reaches highest/lowest values

Recall: qualitative plot of $f : D \to C$



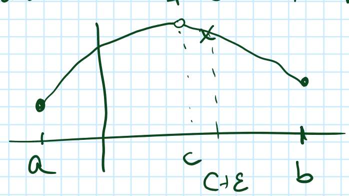
Definitions

$f : [a,b] \to \mathbb{R}$ has a global maximum at c if $f(c) \geq f(x)$, $a \leq x \leq b$.
$a \leq c \leq b$



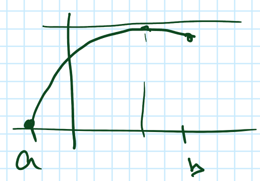
$f : [a,b] \to \mathbb{R}$ has a global minimum at c if $f(c) \leq f(x)$, $a \leq x \leq b$

Ex: f must be defined at minimum/maximum

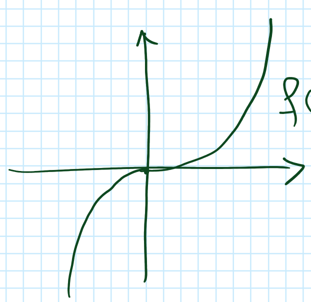


minimum/maximum extremum is either one
extrema is the plural

Th $f : [a,b] \to \mathbb{R}$ f has an extremum at c. and $f'(c)$ exists then $f'(c) = 0$

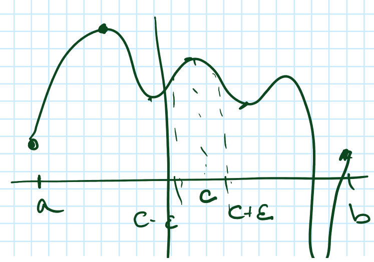


Careful $f'(c) = 0$



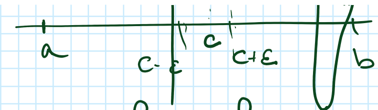
$f(x) = x^3$ $f'(x) = 3x^2$
$f'(0) = 0$

Local extrema



def $f : [a,b] \to \mathbb{R}$
c is a local maximum

def $f:[a,b]\to\mathbb{R}$

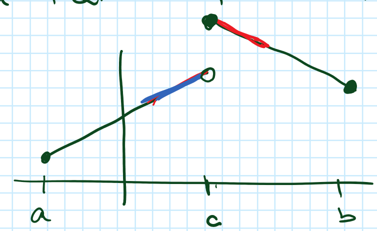


c is a local maximum

if there exists some $\varepsilon>0$ such that $f(c)\geq f(x)$; $c-\varepsilon<x<c+\varepsilon$
(∃)

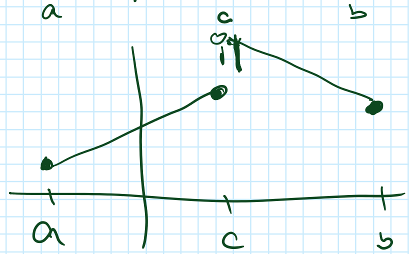
def $f:[a,b]\to\mathbb{R}$ c is a local minimum
if $\exists\ \varepsilon>0$ such that $f(c)\leq f(x)$ $c-\varepsilon<x<c+\varepsilon$

Ex: 1) Graph examples



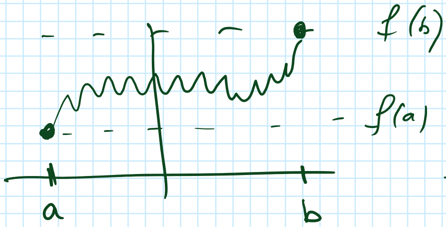
Have a maximum

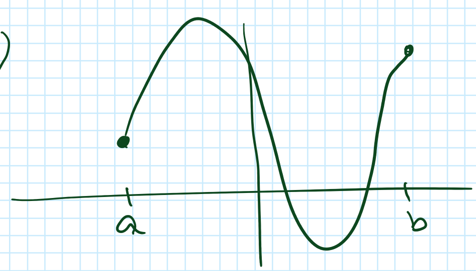
2)



Does not have a maximum

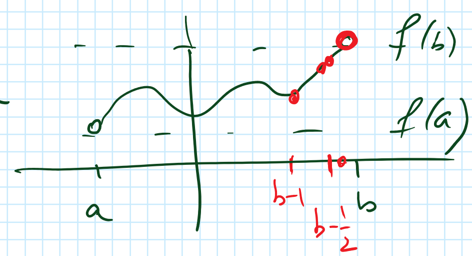
3)





Theorem $f:[a,b]\to\mathbb{R}$ continuous has a maximum and has a minimum.

Ex. $f:(a,b)\to\mathbb{R}$



Introduce idea of interesting points for a function

Critical points = points of discontinuity, extrema of function, i.e. where $f'(c)=0$ or f' does not exist.

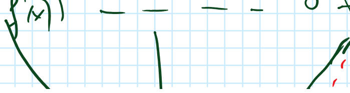
Qualitative ploting of a function:

23. $f(x)=3x^2-4x+2$

$f:\mathbb{R}\to\mathbb{R}$

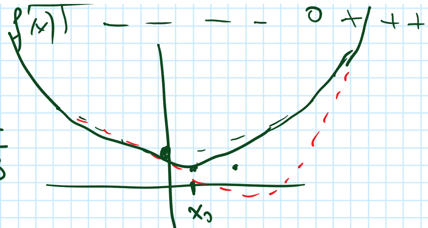
x	$-\infty$		0	$\frac{2}{3}$	∞
$f(x)$	∞	↘ ↘	2	$\frac{2}{3}$	↗ ↗ ∞
$f'(x)$	− −	− −		0	+ + +

$f'(x)=6x-4$



23. $f(x)=3x^2-4x+2$

$f'(x) = 6x - 4$



$f'(x_0) = 0 \Rightarrow x_0 = \frac{2}{3}$

$6x_0 - 4 = 0 \Rightarrow$

$f\left(\frac{2}{3}\right) = 3\,\frac{2^2}{3^2} - 4\,\frac{2}{3} + 2 =$

$= \frac{4}{3} - \frac{8}{3} + 2 = -\frac{4}{3} + 2 = \frac{2}{3}$

23. $f(x) = 3x^2 - 4x + 2$

24. $f(x) = \frac{1}{8}x^3 - \frac{1}{2}x$

25. $f(x) = \frac{x^3}{3} - 9x$

26. $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - 3x^2 + 10$

27. $f(x) = 3x^3 + \frac{3x^2}{2} - 2x$

28. $f(x) = \frac{4x^5}{5} - 3x^3 + 5$

29. $f(x) = x^3 - 4a^2x$

30. $f(x) = x - 5\tan^{-1}x$

31. $f(t) = \frac{t}{t^2+1}$

32. $f(x) = 12x^5 - 20x^3$

33. $f(x) = \frac{e^x + e^{-x}}{2}$

34. $f(x) = \sin x \cos x$

35. $f(x) = \frac{1}{x} + \ln x$

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35. $f(x) = \frac{1}{x} + \ln x \qquad f: (0, \infty) \to \mathbb{R}$

x	0	1	∞
$f(x)$	∞	1	∞
$f'(x)$	– – –	0	+ + + + +

$f'(x) = -\frac{1}{x^2} + \frac{1}{x} = \frac{x-1}{x^2}$

$f'(x_0) = 0 \Rightarrow \quad x_0 = 1$