Tuesday, October 18, 2022 9:29 AM

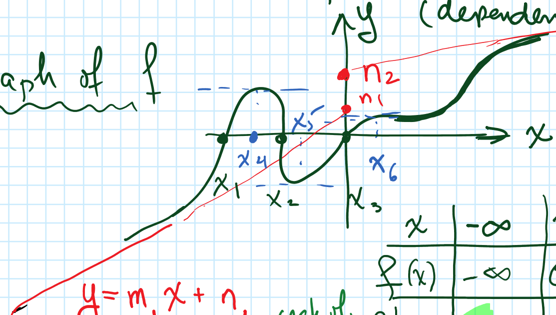
Uses of derivatives: graphing of functions

$f : D \to R$ $\qquad$ D = domain $\quad$ R = (range) co-domain

f is the function, $f(x)$ the values of the function at x (dependent variable, $y = f(x)$)

Graph of f

$f : D \to C$

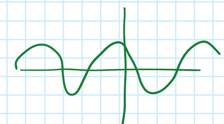


x	$-\infty$	x_1	x_4	x_2	x_5	x_3	x_6	$+\infty$
$f(x)$	$-\infty$	0	+	0	−	0	+	∞
$\nearrow$ or $\searrow$ (rate of change) $f'(x)$	m_1	+	0	−	0	+	0	m_2
$\cup$ or $\cap$ (curvature) $f''(x)$	0	+ ?	−		+		−	0

$f'' > 0 \Rightarrow$ "holds water" "concave up"

rate of increase of the function f is itself increasing

$-3 < -2$



$\sin : \mathbb{R} \to \mathbb{R}$

$[-1, 1]$

? = points at which $f'' = 0$ are called inflection points (change curvature)

$\lim_{x \to \infty} f(x) = \infty \qquad \lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f'(x) = m_2 \qquad \lim_{x \to -\infty} f(x) = m_1$

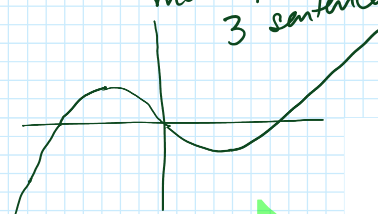
$\lim_{x \to \pm\infty} f''(x) = 0$

Examples

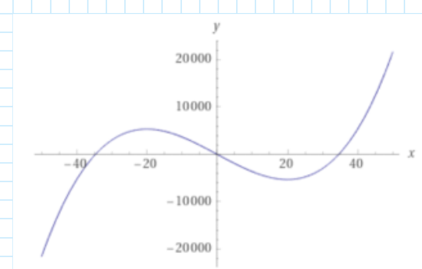
$f(x) = \frac{1}{3} x^3 - 400x$ $\qquad$ Graph f

$f : \mathbb{R} \to \mathbb{R};\ f'(x) = x^2 - 400;\ f''(x) = 2x$ $\quad$ ✓ Well formed math phrase with 3 sentences

x	$-\infty$	$-20\sqrt{3}$	-20	0	20	$20\sqrt{3}$	∞
$f(x)$	$-\infty$ $\nearrow$	$\nearrow$ 0 $\nearrow$	$\sqcup$ $\searrow$	0	$\searrow$ $\sqcup$	$\nearrow$ 0 $\nearrow$	∞
$f'(x)$	∞ +	+	+ 0 −	−400	− 0	+ + +	∞
$f''(x)$	$-\infty$	− $\cap$	− $\cap$ − $\cap$	0	+ $\cup$	+ $\cup$ + $\cup$	∞



$f : (-\infty, \ \to (\ \infty \ \ f' \ \ x^2 - 400$ $\quad$ Malformed math expression



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Obs: f is a polynomial, hence continuous

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f' —— " ——

f'' —— " ——

$f'''(x) = 2$ —— " ——

$$\left(\begin{array}{l} P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \\ \text{for } n=0 \qquad P_0(x) = a_0 \quad \text{a constant} \end{array}\right)$$

$$f(x) = \frac{1}{3}x^3 - 400x = x\left(\frac{1}{3}x^2 - 400\right)$$

$$f(x) = 0 \Rightarrow x_1 = 0; \quad \frac{1}{3}x^2 - 400 = 0 \Rightarrow x^2 = 1200$$

$$\Rightarrow x_{2,3} = \pm 20\sqrt{3} \cong \pm 34$$

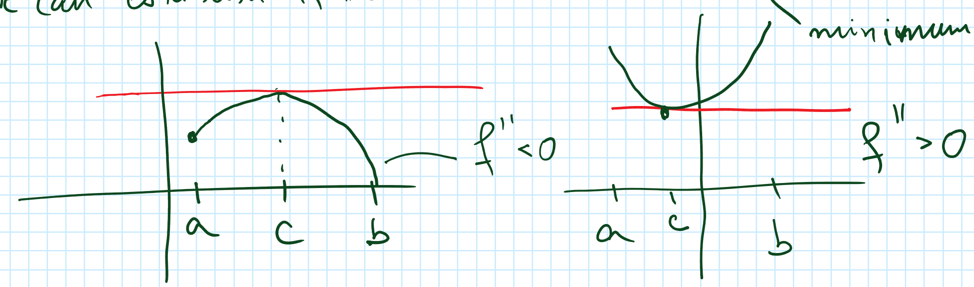
$$f'(x) = x^2 - 400$$

$$f'(x) = 0 \Rightarrow x_{4,5} = \pm 20$$

Recall $f : (a,b) \to \mathbb{R}$ if for some c $a < c < b$

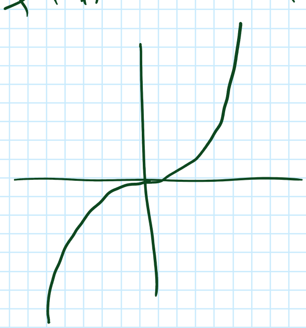
$f'(c) = 0 \Rightarrow c$ is a critical point; it is a local extremum

We can establish if the local extremum is a maximum / minimum by values of f''



Inflection point $\quad f(x) = x^3 \quad f'(x) = 3x^2 \quad f''(x) = 6x$

$f'(x) = 0 \Rightarrow x_1 = 0; \quad f''(x_1) = 0 \qquad x_1$ is an inflection point



E.C. for class participation

Ex: $f(x) = \cos(\sin(x)) + \sin(\cos(x))$

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Compute f': $f = g + h$ (notation) $g(x) = \cos(\sin(x))$
$h(x) = \sin(\cos(x))$

Sum rule: $f' = g' + h'$

Compute g': $g(x) = u(v(x))$ $\left.\begin{array}{l} u(z) = \cos z;\ u'(z) = -\sin z \\ v(x) = \sin(x);\ v'(x) = \cos x \end{array}\right\} \Rightarrow$

Chain rule $g'(x) = u'(z)\, v'(x) = -\sin(\sin(x)) \cos x$

Compute h': $h(x) = s(t(x))$ $s(z) = \sin z$; $s'(z) = \cos z$
$t(x) = \cos x$; $t'(x) = -\sin x$

Chain rule: $h'(x) = s'(z)\, t'(x) = \cos(\cos(x)) \cdot (-\sin x)$

$\cancel{\cos(\cos(x)) \sin x}$