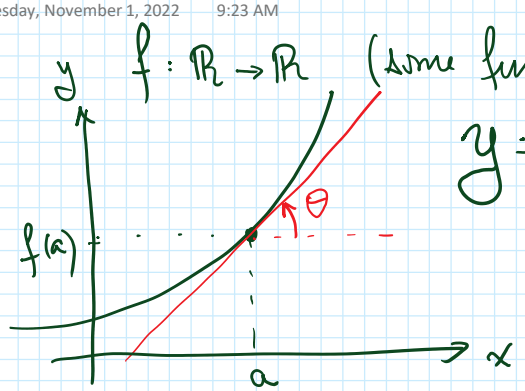
Linear Approximation

Tuesday, November 1, 2022 9:23 AM



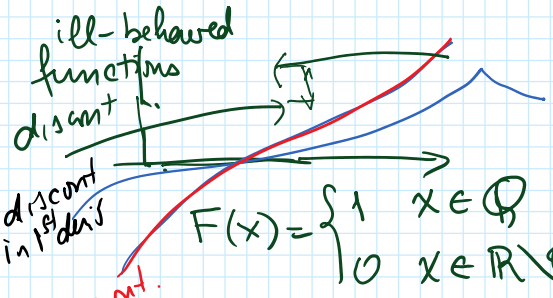
$f: \mathbb{R} \to \mathbb{R}$ (some function)

$$y = f'(a)(x-a) + f(a)$$

Tangent line

$f'(a)$ = slope

$f'(a) = \tan\theta$



$$F(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

$\mathbb{Q}$ = set of rationals

$\mathbb{R} \setminus \mathbb{Q}$ = set of reals with rationals removed

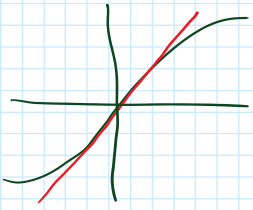
$$y(x) = f'(a)(x-a) + f(a) = L(x)$$

↳ "The linear approximation of f at a"

Ex: $f(x) = \sin x$. Construct linear approximant at $a = 0, \ a = \frac{\pi}{6}, \ a = \frac{\pi}{4}, \ a = \frac{\pi}{3}$

$$L(x) = f'(a)(x-a) + f(a) = (\cos a)(x-a) + \sin a$$

$a = 0 \Rightarrow \quad L(x) = x$

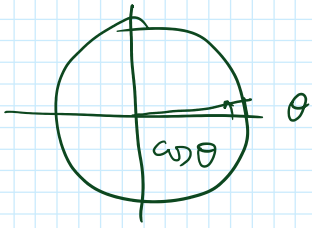


0 $\frac{\sqrt{3}}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{1}{2}$ 1

0 $\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$ 1

$a = \frac{\pi}{6} \Rightarrow \quad L(x) = \left(\cos\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right) + \sin\frac{\pi}{6} \Rightarrow$

$$L(x) = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}$$



$a = \frac{\pi}{4} \Rightarrow \quad L(x) = \left(\cos\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) + \sin\frac{\pi}{4} \Rightarrow$

$$L(x) = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) + \frac{\sqrt{2}}{2}$$

$a = \frac{\pi}{3} \Rightarrow \quad L(x) = \cos\frac{\pi}{3}\left(x - \frac{\pi}{3}\right) + \sin\frac{\pi}{3} \Rightarrow$

$$L(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$$

$L(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$a = \frac{\pi}{2} \Rightarrow \quad L(x) = 0 \cdot \left(x - \frac{\pi}{2}\right) + 1$