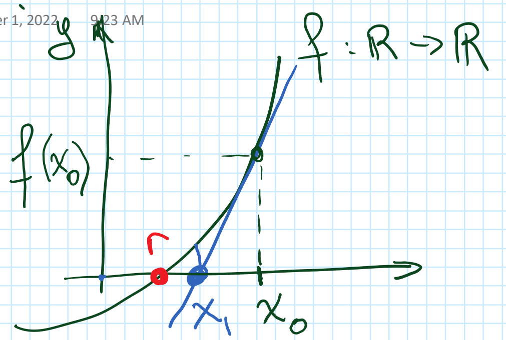
Tuesday, November 1, 2022 9:23 AM

Newton method & l'Hôpital rule



$$\boxed{f(x) = 0}$$

$$y(x) = f'(x_0)(x - x_0) + f(x_0)$$

$$0 = f'(x_0)(x_1 - x_0) + f(x_0)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Some equations can be solved through formula

$$mx + a = b \Rightarrow$$

$$x = \frac{b-a}{m} \quad (m \neq 0)$$

$$Ax^2 + Bx + C = 0 \Rightarrow$$

$$x_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$Ax^3 + Bx^2 + Cx + D = 0$$

$$\sin x + x^2 + 1 = 0$$

$\{x_n\}_{n \in \mathbb{N}}$ sequence of approximants

$$\lim_{n \to \infty} x_n = r$$

Newton's method $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \qquad f' \neq 0$

Ex:

$$x^2 = a \Rightarrow x^2 - a = 0$$

$$f(x) = x^2 - a \qquad f(x_n) = x_n^2 - a$$

Newton's method $\quad x_{n+1} = x_n - \dfrac{x_n - a}{2x_n}$

$$x_{n+1} = x_n - \left(\frac{x_n}{2} - \frac{a}{2x_n}\right) = \frac{x_n}{2} + \frac{a}{2x_n} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

Heron's formula

$\sqrt{2}$

$$x_0 = \frac{3}{2}$$

$$x_1 = \frac{1}{2}\left(\frac{3}{2} + \frac{2 \cdot 2}{3}\right) = \frac{1}{2} \cdot \frac{17}{6} = \frac{17}{12} = 1$$

$$x_2 = \frac{1}{2}\left(\frac{17}{24} + \frac{12}{17}\right) =$$

Please complete at home !

... try to learn a lesson. ☺

$\sqrt{3}$

l'Hôpital rule

(Really an important concept)

Recall definition of function derivative

$f : \mathbb{R} \to \mathbb{R}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \overset{?}{=} \cancel{\frac{\lim_{h\to 0} f(x+h)-f(x)}{\lim_{h\to 0} h}}$$

"$\frac{0}{0}$" is called an "indeterminancy"

Observe

$g(x) = x$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{g(x+h)-g(x)} = \lim \frac{f(x+h)-f(x)}{h}$$

think

Make g be some general function

$$\lim_{h\to 0} \boxed{\frac{f(x+h)-f(x)}{g(x+h)-g(x)}} = \lim_{h\to 0} \left(\frac{\frac{f(x+h)-f(x)}{h}}{\frac{g(x+h)-g(x)}{h}} \right) = \circ\!-$$

$$\circ\!- = \frac{\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}{\lim_{h\to 0} \frac{g(x+h)-g(x)}{h}} = \frac{f'(x)}{g'(x)}$$

l'Hôpital's rule

If $\frac{0}{0}$ indeterminancy

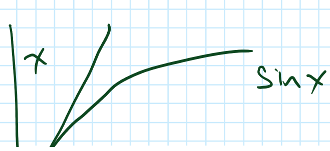
$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

if f', g' are defined

$\lim_{x\to a} f(x) = 0$

$\lim g(x) = 0$

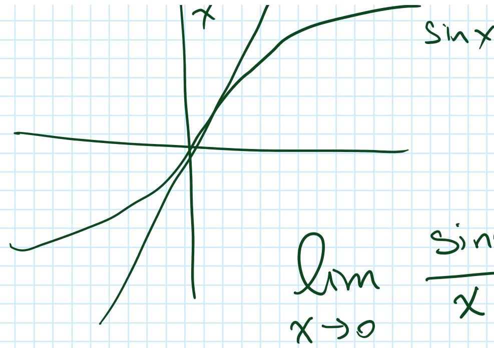
Ex:



$x \to a$

$$\lim_{x\to a} g(x) = 0$$

Ex:



$$\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{(\sin x)'}{(x)'} = \lim_{x\to 0}\frac{\cos x}{1} = 1$$

Ex:

17. $\lim\limits_{x\to 2}\dfrac{x^2-2x}{x^2-6x+8}$

18. $\lim\limits_{x\to -1}\dfrac{x^4+x^3+2x+2}{x+1}$

19. $\lim\limits_{x\to 1}\dfrac{x^2+2x}{x+3}$

20. $\lim\limits_{x\to 0}\dfrac{e^x-1}{2x+5}$

21. $\lim\limits_{x\to 1}\dfrac{\ln x}{4x-x^2-3}$

22. $\lim\limits_{x\to 0}\dfrac{e^x-1}{x^2+3x}$

23. $\lim\limits_{x\to \infty}\dfrac{3x^4-x^2}{6x^4+12}$

24. $\lim\limits_{x\to \infty}\dfrac{4x^3-2x^2+6}{\pi x^3+4}$

25. $\lim\limits_{x\to e}\dfrac{\ln x-1}{x-e}$

26. $\lim\limits_{x\to 1}\dfrac{4\tan^{-1}x-\pi}{x-1}$

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$$\lim_{x\to 2}\frac{2x-2}{2x-6} = -1$$

$$\lim_{x\to -1}\frac{4x^3+3x^2+2}{1} = 1$$

$$\lim_{x\to 1}\frac{x^2+2x}{x+3} = \frac{3}{4}$$ (direct substitution)

$$\cancel{\lim_{x\to 1}\frac{2x+2}{1} = 4}$$

Check $\left(\frac{0}{0}\right)$

21. $\left(\frac{0}{0}\right)$ ✓ $\lim\limits_{x\to 1}\dfrac{\frac{1}{x}}{4-2x} = \dfrac{1}{2}$ ✓

25. $\lim\limits_{x\to e}\dfrac{\frac{1}{x}}{1} = \dfrac{1}{e}$

$\left(\frac{0}{0}\right)$ ✓ 26. $\lim\limits_{x\to 1}\dfrac{\frac{4}{1+x^2}}{1} = 2$