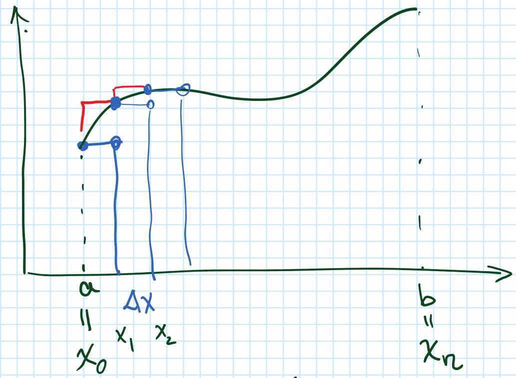
Riemann sums, Area under the curve

Tuesday, November 8, 2022 10:38 AM



$f : [a,b] \to \mathbb{R}$

$x_1 = x_0 + \Delta x$

$x_2 = x_1 + \Delta x = x_0 + 2\Delta x$

...

$x_n = x_{n-1} + \Delta x = x_0 + n\,\Delta x$

Left area rule $= (\Delta x)\left(f(x_0) + f(x_1) + \dots + f(x_{n-1})\right)$

Righ area rule $= (\Delta x)\left(f(x_1) + f(x_2) + \dots f(x_n)\right)$

Mid —"— $= (\Delta x)\left(f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \dots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$

→ Riemann Sums → HW11.

Σ - Sigma (σ) notation

Left area rule $= (\Delta x)\sum_{k=0}^{n-1} f(x_k)$

Right area rule $= (\Delta x)\sum_{k=1}^{n} f(x_k) = (\Delta x)\sum_{l=1}^{n} f(x_l)$

Mid —"— $= (\Delta x)\sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$

— Sub-intervals are not of equal length

7 9 15 19

Ex. 5.1.7

7. Suppose you want to approximate the area of the region bounded by the graph of $f(x) = \cos x$ and the x-axis between $x = 0$ and $x = \frac{\pi}{2}$. Explain a possible strategy.

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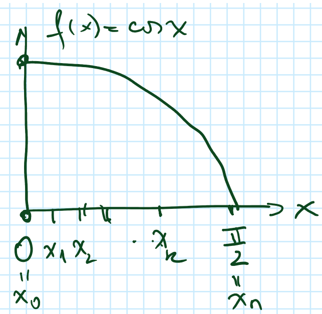
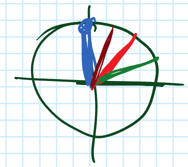
Equidist. subint

$x_k = x_0 + k\,\Delta x$, $k = 0, 1, \dots, n$

$x_0 = 0 \quad \Delta x = \frac{b-a}{n} = \frac{\pi}{2n}$

n = no. of subintervals





$\text{Right}(\text{Area}) \simeq (\Delta x)\sum_{k=1}^{n} f(x_k)$

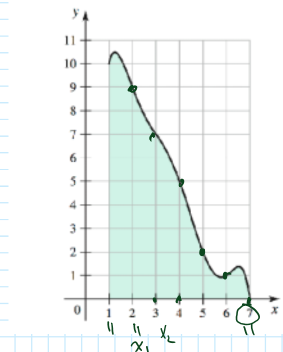
$\text{Left}(\ ) \simeq (\ )\sum^{n} f(x_{k-1})$

9. Approximating area from a graph Approximate the area of the region bounded by the graph (see figure) and the x-axis by dividing the interval

$$\left(\text{Area}\right) = (\Delta x)\sum_{k=1}^{n} f(x_k)$$

$$\left|\begin{matrix}\text{Left}\\ \text{Area}\end{matrix}\right| \approx (\Delta x) \sum_{k=1}^{n} f(x_{k-1})$$

9. Approximating area from a graph Approximate the area of the region bounded by the graph (see figure) and the x-axis by dividing the interval $[1, 7]$ into $n = 6$ subintervals. Use a left and right Riemann sum to obtain two different approximations.



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$\boxed{x_0 \qquad x_6}$

19 27 31
40 , 45

Ex. 5.1.9

$$\left(\text{Area}\right) = \Delta x \sum_{k=1}^{6} f(x_k)$$

$$= \Delta x \left(f(x_1) + f(x_2) + \ldots + f(x_6) \right)$$

$$= 1 \cdot (9 + 7 + 5 + 2 + 1 + 0)$$

$$24$$

5.1.19. $v(t) = \dfrac{1}{2t+1} \; \dfrac{m}{s}$ $\quad 0 \le t \le 8 ;\; n = 4$

$x(0) =$ "choice of origin" $= 0$

19. $v = \dfrac{1}{2t+1}$ (m/s), for $0 \le t \le 8$; $n = 4$

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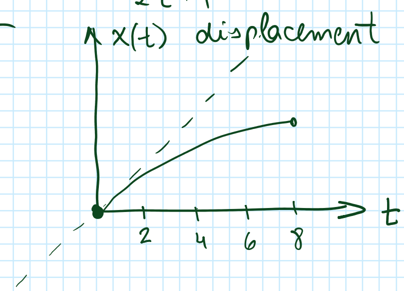

$$x'(t) = v(t)$$

$$x'(0) = v(0) = 1$$

$$x'(2) = v(2) = \frac{1}{5}$$

$$x'(4) = v(4) = \frac{1}{9}$$

$$x'(6) = v(6) = \frac{1}{13}$$

$$x'(8) = v(8) = \frac{1}{17}$$

Over $0 \le t \le 2$

$$x(2) = x(0) + \Delta t \cdot v(0)$$

$$= 0 + 2 \cdot 1 = \underline{2}$$

$$x(4) = x(2) + \Delta t \cdot v(2)$$

$$= 2 + 2 \cdot \frac{1}{5}$$