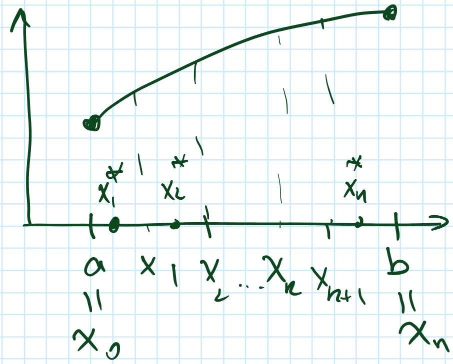
Fundamental Theorem of Calculus

Tuesday, November 15, 2022 10:19 AM

Recall



$f : [a,b] \to \mathbb{R}$

f continuous

x_k^* an arbitrary pt.

$x_{k-1} \le x_k^* \le x_k$

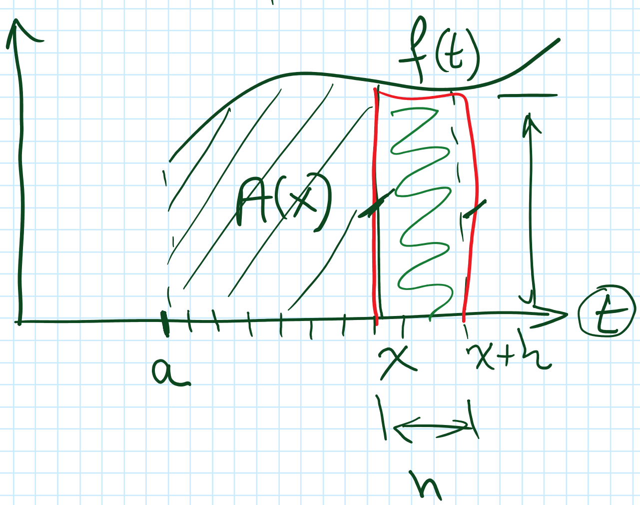
"Size of partition" $= \max_{1 \le k \le n} |x_k - x_{k-1}|$

Riemann sum $= \displaystyle\int_a^b f(x)\,dx = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*)(x_k - x_{k-1})$

— Area function $A(x)$ = "signed area from the graph to the x-axis from a to x"

$$A(x) = \int_a^{x} f(t)\,dt$$

$$A(x+h) = \int_a^{x+h} f(t)\,dt$$



$$A(x+h) - A(x) \cong h\,f(x)$$

$$\frac{A(x+h) - A(x)}{h} \cong f(x)$$

Take limit $\displaystyle\lim_{h \to 0} \frac{A(x+h) - A(x)}{h} = f(x)$

$$A'(x) = f(x)$$

$$F'(x) = f(x)$$

$$\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$$