MATH 231.FA22 Practice Makeup Test 2

Solve the following exercises concisely explaining all solution steps. Answers without presentation of justification of solution steps are not awarded credit. (35 minutes, 1.5 course points per multiple choice question, 4 course points per free-form question)

1. What is the derivative of the function

$$f(x) = \frac{\sin(x) + \cos(y)}{\cos(x) + \sin(y)}?$$

   a) $f'(x) = \frac{\sin(x)\cos(y) + \cos(x)\sin(y)}{(\cos(x) + \sin(y))^2}$.

   b) $f'(x) = \frac{\sin(x)\cos(x) + \cos(y)\sin(y) - 1}{(\cos(x) + \sin(y))^2}$.

   c) $f'(x) = \frac{\sin(x)\cos(y) + \cos(x)\sin(y) + 1}{(\cos(x) + \sin(y))^2}$.

   d) $f'(x) = \frac{\sin(x)\cos(y) + \cos(x)\sin(y) + 1}{(\cos(x))^2 + (\sin(y))^2}$.

2. Identify the three functions f, g, h in the composite function

$$u(t) = (f \circ g \circ h)(t) = f(g(h(t))) = \cos^4(t^2 + 1).$$

   a) $f(x) = t^4$, $g(x) = \cos(t)$, $h(t) = t^2 + 1$.

   b) $f(x) = x^4$, $g(x) = \cos(x)$, $h(x) = x^2 + 1$.

   c) $f(t) = \cos(t)$, $g(t) = t^4$, $h(t) = t^2 + 1$.

   d) $f(u) = u^4$, $g(v) = \cos(v)$, $h(x) = t^2 + 1$.

3. Find the derivative $y'(x)$ of the function $y(x)$ defined implicitly by

$$\cos(y^2) + x = a^y, a > 0.$$

4. Determine the critical points of

$$f(x) = 10^x \left(\ln(10^x) - x\right).$$

5. An object is moving vertically according to the distance function $s(t) = t^3 - 8t^2 - 12t - 3$. Determine the intervals over which the object is moving down.