Monday, September 19, 2022 10:34 AM

Notes: 1) You must present the justification for solution steps, final answer
2) Solutions without justification are not credited, even if correct
3) Organize your solution neatly & cogently

Solutions

1. Denote $L = \lim_{x\to 1} \frac{1-x^4}{x^2-1} = \lim_{x\to 1} f(x)$ (notation)

For $x \neq 1$ rewrite $f(x) = \frac{1-x^4}{x^2-1} = \frac{(1-x^2)(1+x^2)}{x^2-1}$ ($a^2-b^2=(a-b)(a+b)$ identity)

Simplify $f(x) = -(1+x^2)$ for $x \neq 1$

$L_+ = \lim_{x\to 1_+} f(x) = -2$ (direct substitution of x)

$L_- = \lim_{x\to 1_-} f(x) = -2$ —"— (one-sided limits theorem)

Since one-sided limits exist and are equal it results that L exists and $L=-2$.

2. $f(x) = \begin{cases} \frac{x^2-5x+6}{x-3} & \text{if } x\neq 3 \\ a & \text{if } x=3 \end{cases}$

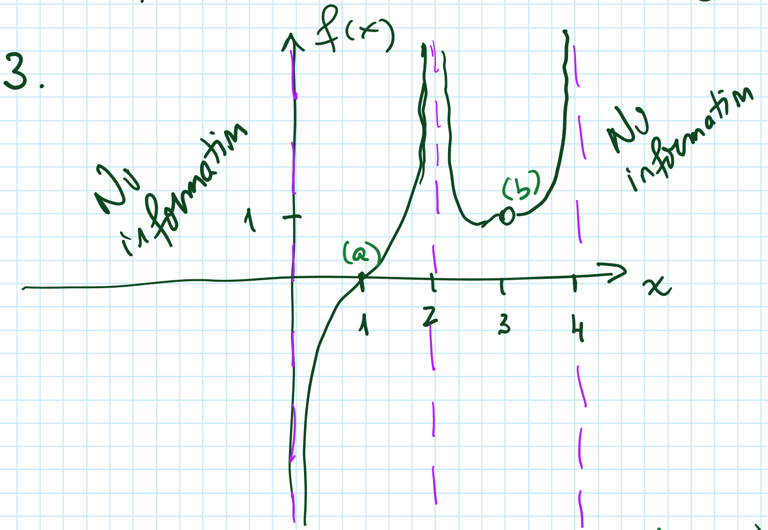
For $x\neq 3$ $f(x) = \frac{P(x)}{Q(x)}$ is ratio of continuous functions, hence f is continuous (Theorem on continuity of ratios)

$f(x)$ continuous at $x=3$ if $\lim_{x\to 3} f(x) = f(3)$ (Definition of continuity)

For $x\neq 3$ rewrite $f(x) = \frac{(x-3)(x-2)}{x-3} = x-2$

Compute $\lim_{x\to 3} f(x) = 1$, hence $a = 1$ for f to be continuous everywhere.

3.



✓ (a) $f(1) = 0$

(b) $f(3)$ undefined

(c) $\lim_{x\to 3} f(x) = 1$

✓ (d) $\lim_{x\to 0^+} f(x) = -\infty$

(e) $\lim_{x\to 2} f(x) = \infty$

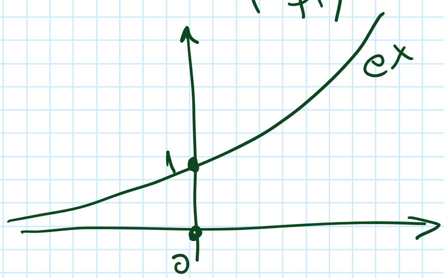
(f) $\lim_{x\to 4^-} f(x) = \infty$

(d) $\Rightarrow$ Vertical asymptote at 0 (right)

(e) $\Rightarrow$ —— " —— at 2 (both)

(f) $\Rightarrow$ —— " —— at 4 (left)

4. $f(x) = \dfrac{2e^x + 3}{e^x + 1}$



Plot of e^x

For $x \to \infty$ rewrite $f(x) = \dfrac{e^x(2 + 3e^{-x})}{e^x(1 + e^{-x})} = \dfrac{2 + 3e^{-x}}{1 + e^{-x}}$ $(e^x \neq 0)$

As $x \to \infty$ $e^{-x} \to 0$, hence $\lim_{x\to\infty} f(x) = 2$

As $x \to -\infty$ $e^{x} \to 0$, hence $\lim_{x\to\infty} f(x) = 3$

5. $f(x) = e^x (x^2 - x + 1) = g(x)\,h(x)$ (Notation)

$f' = g'h + gh'$ (Product rule)

$f'' = (g'h + gh)' = (g'h)' + (gh')'$ (Sum rule)

(Product rule

$$f'' = (g'h + gh')' = (g'h)' + (gh')'$$

$$f'' = g''h + g'h' + g'h' + gh''$$ (Product rule

$$f'' = g''h + 2g'h' + gh''$$

$$g'(x) = e^x \; ; \; g''(x) = e^x$$ (Diff. table : exponential)

$$h'(x) = 2x - 1 \; ; \; h''(x) = 2$$ (Diff. table : polynomial)

Substitute

$$f'(x) = e^x\left(x^2 - x + 1 + 2x - 1\right) = e^x\left(x^2 + x\right)$$

$$f''(x) = e^x\left(x^2 - x + 1 + 4x - 2 + 2\right) = e^x\left(x^2 + 3x + 1\right)$$