MATH 231.FA22 PRACTICE TEST 1

Solve the following exercises explaining all solution steps. (45 minutes)

1. State whether the limit

$$\lim_{x \to 1} \frac{1 - x^4}{x^2 - 1}$$

exists, and, if so, compute the limit.

2. Find the constant a such that

$$f(x) = \begin{cases} \dfrac{x^2 - 5x + 6}{x - 3} & \text{if } x \neq 3 \\ a & \text{if } x = 3 \end{cases}$$

is continuous for all x.

3. Sketch the graph of $f(x)$ knowing that:

$$f(1) = 0, \quad f(3) \text{ undefined}, \quad \lim_{x \to 3} f(x) = 1, \quad \lim_{x \to 0^+} f(x) = -\infty, \quad \lim_{x \to 2} f(x) = \infty, \quad \lim_{x \to 4^-} f(x) = \infty \ .$$

4. Determine the limits at $\pm\infty$ of the function

$$f(x) = \frac{2e^x + 3}{e^x + 1} \ .$$

5. Compute the first and second derivative of

$$f(x) = e^x(x^2 - x + 1).$$