MATH 231.FA22 Practice Test 2

Solve the following exercises explaining all solution steps. (45 minutes)

1. Compute the derivative $s'(t)$ of

$$s(t) = \frac{t^{4/3}}{e^t}$$

exists, and, if so, compute the limit.

2. Compute the derivative $y'(x)$ of

$$y(x) = \frac{a\sin x + b\cos x}{a\sin x - b\cos x}$$

where a, b are nonzero constants..

3. The distance of an object from its starting point is $s(t) = -6t^3 + 36t^2 - 54t$. Determine the acceleration of the object whenever its velocity is zero for $0 \leqslant t \leqslant 4$.

4. Consider $y(x)$ defined implicitly by

$$(x+y)^{2/3} = y.$$

Compute $y'(x)$ and the slope of $y(x)$ at $(x, y) = (4, 4)$.

5. The hypotenuse of an isosceles right triangle decreases in length at rate $4m/s$.

   a) At what rate is the area of the triangle increasing when the legs are $5m$ long?

   b) At what rate are the lengths of the legs of the triangle changing?

   c) At what rate is the area of the triangle changing when the area is $4m^2$?