R01 - Model solutions

Monday, August 22, 2022 2:25 PM

§2.3. Limit techniques

2.3.72. Let $g(x) = \begin{cases} 5x-15 & \text{if } x<4 \\ \sqrt{6x+1} & \text{if } x \geq 4 \end{cases}$

Concisely, neatly state problem

Compute: (a) $\lim_{x \to 4^-} g(x)$ (b) $\lim_{x\to 4^+} g(x)$ (c) $\lim_{x\to 4} g(x)$

Show understanding of notation

Solution: (a) $x \to 4^-$ implies $x \to 4$ and $x<4$, hence

$g(x) = 5x-15$ $\lim_{x\to 4^-} g(x) = 5\cdot 4 - 15 = 5$

find solution

Answer with no credit would be (a) $\lim_{x\to 4^-} g(x) = 5$.

Yes, the answer is correct, but we are grading your *understanding*, not the appearance of a correct answer.

(b) $x \to 4^+$ implies $x \to 4$ and $x>4$ hence

$g(x) = \sqrt{6x+1}$ $\lim_{x\to 4^+} g(x) = \sqrt{6\cdot 4+1} = 5$

(c) Since $\lim_{x\to 4^+} g(x) = \lim_{x\to 4^-} g(x)$, (the one-sided limits are equal), then $\lim_{x\to 4} g(x)$ exists and

$\lim_{x\to 4} g(x) = 5$ (Theorem 2.1)

State theory that supports your conclusion.

2.3.87 $f(x) = \begin{cases} \dfrac{x^2-5x+6}{x-3} & \text{if } x \neq 3 \\ a & \text{if } x=3 \end{cases}$

Concisely, neatly state problem

Find a such that $\lim_{x\to 3} f(x) = f(3)$.

Solution. Factor $x^2-5x+6 = (x-3)(x-2)$ and replace in definition of $f(x)$ of $x \neq 3$ (State solution procedure)

$$f(x) = \begin{cases} \dfrac{(x-3)(x-2)}{x-3} = x-2 & \text{if } x \neq 3 \\ a & \text{if } x = 3 \end{cases}$$

$\lim_{x\to 3^-} f(x) = 2$ (limit of linear function $x-2$)

$\lim_{x\to 3^+} f(x) = 2$ (—"—)

Choose $a=2$ such that $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

(Theorem 2.1) State theory and result.