R02 Model solutions

Monday, August 29, 2022 5:04 PM

Oblique asymptote problem §2.5.51 $f(x) = \dfrac{x^2-3}{x+6}$

a) Find the slant asymptote.

Theory: slant asymptote is the line approached by graph of $f(x)$ as $x \to \pm\infty$ (State theory, definitions)

Carry out long division (State calculation procedure)

$$\begin{array}{r|l} & x - 6 \\ \hline x+6 & x^2 + 0.x - 3 \\ & x^2 + 6x \\ \hline = & -6x - 3 \\ & -6x - 36 \\ \hline & 33 \end{array}$$

Hence $f(x) = \dfrac{x^2-3}{x+6} = x - 6 + \dfrac{33}{x+6}$

Verify by bringing to common denominator

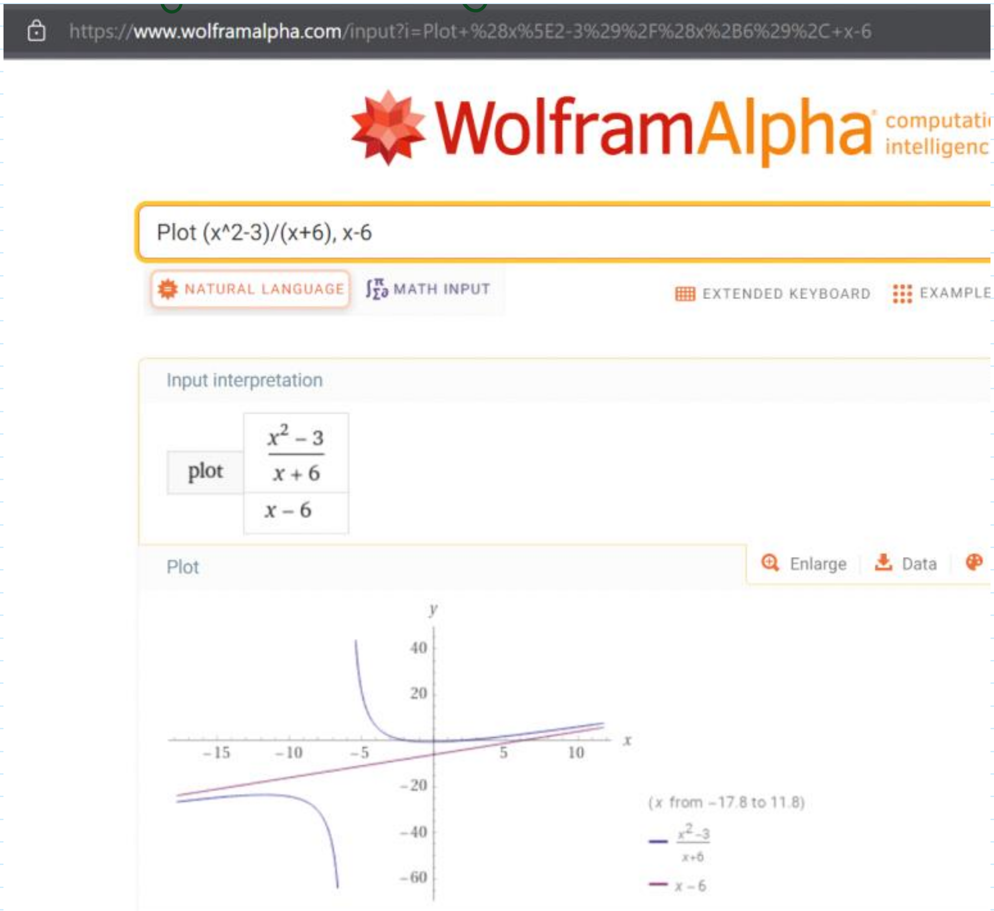
$x - 6 + \dfrac{33}{x+6} = \dfrac{x^2 - 36 + 33}{x+6} = \dfrac{x^2-3}{x+6}$ ✓ (Check for errors)

$f(x) = x - 6 + \dfrac{33}{x+6} = l(x) + r(x)$ (Introduce notation)

As $x \to \pm\infty$ $\lim_{x\to\pm\infty} r(x) = 0$, or $r(x)$ becomes negligible for large x

Hence, for large x, $f(x)$ behaves like $l(x) = x - 6$.

Verify computationally in your browser at www.wolframalpha.com



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b. Find vertical asymptotes.

$$f(x) = \frac{x^3-3}{x+6} = \frac{p(x)}{q(x)}$$ (Introduce notation for numerator/denominator)

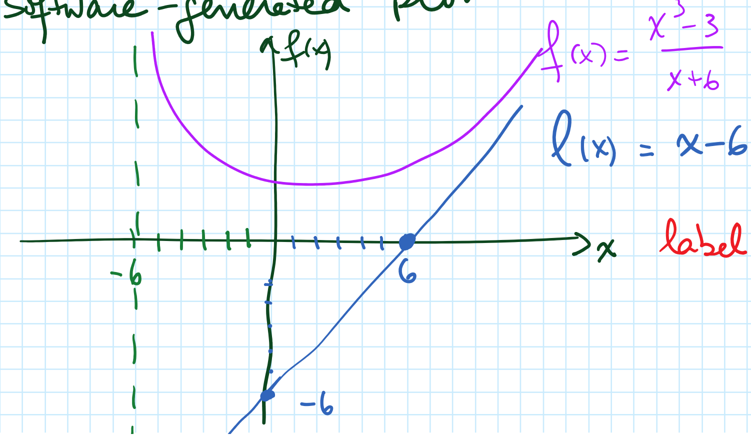
f(x) has a vertical asymptote where denominator is zero (State theory)

$q(x) = 0 \Rightarrow x+6=0 \Rightarrow x=-6$ is a vertical asymptote as seen in above plot.

c. Sketch f(x) using software and by hand

See above software-generated plot.

By hand:



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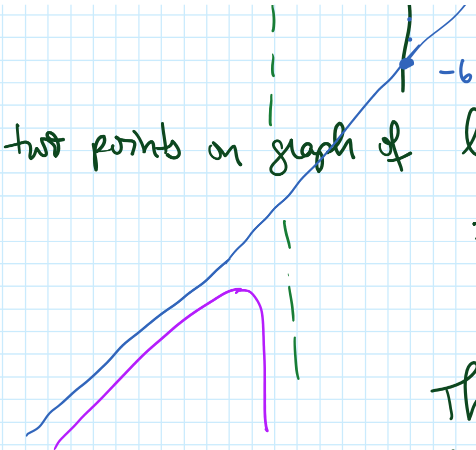

Find two points on graph of $l(x)$: $l(0) = -6$ $l(6) = 0$

For $x < -6$

$$f(x) = l(x) + \frac{33}{x+6} = l(x) + \text{"something negative"}$$

Thus graph of $f(x)$ below that of $l(x)$, $f(x) < l(x)$

(State reasoning for hand drawn graph)

For $x > -6$ $f(x)$ above $l(x)$ $f(x) > l(x)$.