R11 Model Solutions

Monday, October 31, 2022 10:43 AM

4.7.83. $\lim_{x\to 0} (x+\cos x)^{\frac{1}{x}}$

Step 1: Identify type of indeterminancy; $f(x) = x+\cos x$; $g(x) = \frac{1}{x}$

$\lim_{x\to 0} (x+\cos x) = 1$ $\quad \lim_{x\to 0_+} \frac{1}{x} = \infty$ $\quad \lim_{x\to 0_-} \frac{1}{x} = -\infty$

Indeterminancy of form $1^{\pm\infty}$ $\Rightarrow$ take ln

Step 2: Analyze limit $L = \lim_{x\to 0} g(x) \ln f(x) = \lim_{x\to 0} \frac{\ln(x+\cos x)}{x}$

Step 3: Check if l'Hopital can be applied

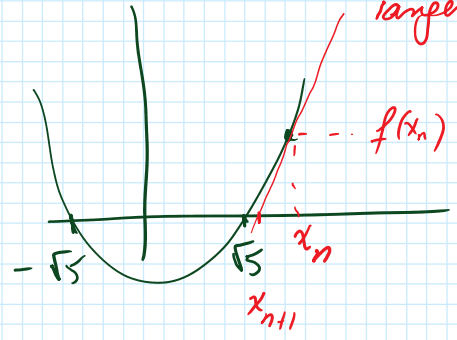
$\lim_{x\to 0} \ln(x+\cos x) = 0$, $\lim_{x\to 0} x = 0$. Yes, $\frac{0}{0}$ indeterminancy with differentiable functions $\Rightarrow$ l'Hopital can be applied.

Step 4: Apply l'Hôpital

$$L = \lim_{x\to 0} \frac{\frac{1-\sin x}{x+\cos x}}{1} = \lim_{x\to 0} \frac{1-\sin x}{x+\cos x} = 1$$ (Apply Exponential)

$$\Rightarrow \boxed{\lim_{x\to 0} (x+\cos x)^{\frac{1}{x}} = e}$$

4.9.8. Newton method formula for $f(x) = x^2 - 5$.



Tangent line: $y = f'(x_n)(x - x_n) + f(x_n)$

Choose x_{n+1} to be intersect of tangent with x-Axis

$\Rightarrow 0 = f'(x_n)(x_{n+1} - x_n) + f(x_n) \Rightarrow$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 5}{2x_n} = x_n - \frac{x_n}{2} + \frac{5}{2x_n} =$$

$$= \frac{x_n}{2} + \frac{5}{2x_n} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right)$$