R13 Working with integrals

Saturday, November 19, 2022 12:28 PM

Note: Final examination can include these topics.
Multiple model solutions are provided.

29–62. Definite integrals *Evaluate the following integrals using the Fundamental Theorem of Calculus.*

29. $\int_0^2 4x^3\,dx$

30. $\int_0^2 (3x^2+2x)\,dx$

31. $\int_1^8 8x^{1/3}\,dx$

32. $\int_1^{16} x^{-5/4}\,dx$

33. $\int_0^1 (x+\sqrt{x})\,dx$

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29. $I = \int_0^2 4x^3\,dx$

Step 1: identify integrand: $f(x) = 4x^3$
integration variable: x

Step 2: Find anti-derivative

$F(x) = \int 4x^3\,dx = x^4 + C$

Check: $F'(x) = 4x^3 = f(x)$ ✓

Step 3: Apply Fundamental Theorem of Calculus (FTC)

$I = F(2) - F(1) = 2^4 - 1 = 15.$

30. $I = \int_0^2 (3x^2+2x)\,dx = \int_0^2 f(x)\,dx$

Step 1: $f(x) = 3x^2+2x$

Step 2: $F(x) = \int (3x^2+2x)\,dx = x^3 + x^2 + C$

$F'(x) = 3x^2+2x = f(x)$ ✓

Step 3: $I = F(2) - F(0) = 2^3 + 2^2 - 0^3 - 0^2 = 12.$

31. $I = \int_1^8 8x^{1/3}\,dx$

1) $f(x)' = 8x^{1/3}$

2) $F(x) = 6x^{4/3} + C$

3) $I = F(8) - F(1) = 6\left(8^{4/3} - 1\right) = 6 \cdot 15 = 90.$

20. $\int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx$

21. $\int (x^2+x)^{10}(2x+1)\,dx$

22. $\int \frac{1}{10x-3}\,dx$

23. $\int x^3(x^4+16)^6\,dx$

24. $\int \sin^{10}\theta\cos\theta\,d\theta$

25. $\int \frac{dx}{\sqrt{36-4x^2}}$

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5.5.20. $F(x) = \int \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx$

Step 1) Identify possible substitution

$u(x) = \sqrt{x}$

Step 2) Differentiate:

$u'(x) = \frac{du}{dx} = \frac{1}{2\sqrt{x}}$; $dx = 2\sqrt{x}\,du$

Step 3) Substitute

$I = \int \frac{(u+1)^4}{2\sqrt{x}}\,2\sqrt{x}\,du$

Find the primitive

$I = \int \frac{\quad}{2\sqrt{x}}$

Step 4) Compute primitive

$$I = \frac{1}{5}(u+1)^5 + C$$

Step 5) Replace substitution

$$F(x) = \frac{1}{5}(\sqrt{x}+1)^5 + C$$

Step 6) Verify

$$F'(x) = (\sqrt{x}+1)^4 \cdot \frac{1}{2\sqrt{x}} \quad \checkmark$$

5.5.21 $F(x) = \int (x^2+x)^{10}(2x+1)\,dx$

1) $u(x) = x^2+x$

2) $u'(x) = \frac{du}{dx} = 2x+1 \Rightarrow du = (2x+1)dx$

3) $I = \int u^{10}\,du = \frac{1}{11}u^{11} + C$

4) $F(x) = \frac{1}{11}(x^2+x)^{11} + C$

5) $F'(x) = (x^2+x)^{10}(2x+1)$ ✓