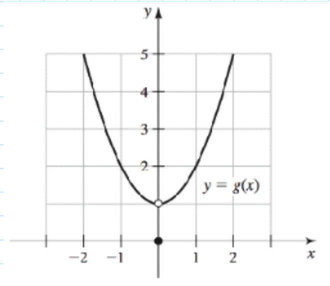
HW01 Solution (For reference & test preparation)

Tuesday, September 6, 2022 12:14 PM

2.2.4. Identifying limits from function graphs

a) $g(0)$



$g(0)$ is the value of function $g: \mathbb{R} \to \mathbb{R}$ at $x=0$.

Per graph convention, a filled-in dot represents a defined function value, hence $g(0)=0$.

b) $\lim\limits_{x \to 0} g(x)$

Per graph convention, g is defined on an interval around 0, hence a limit can be taken

$$\lim_{x \to 0} g(x) = 1$$

c) $g(1)$

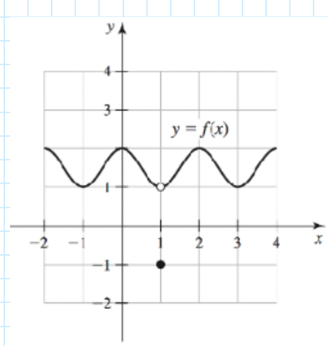
Per graph convention, a continuous line indicate a function defined via an interval, and $g(1)=2$

d) $\lim\limits_{x \to 1} g(x)$

g is defined on an interval around $x=1$

and $\lim\limits_{x \to 1} g(x) = 2$

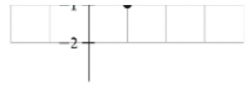
2.2.5 a) $f(1)$



Per graph convention $f(1)=-1$

b) $\lim\limits_{x \to 1} f(x)$

Per graph convention f is defined on intervals $(0,1)$ and $(1,2)$. One-sided limits can defined on intervals $(0,1)$ and $(1,2)$. One-sided limits can be taken $\lim_{x^+ \to 1} f(x) = 1$ $\lim_{x^- \to 1} f(x) = 1$ and are equal hence $\lim_{x \to 1} f(x) = 1$



c) $f(0)$

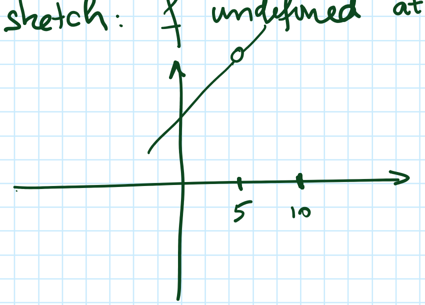
Per graph convention, $f(0) = 2$

d) $\lim_{x \to 0} f(x)$

Per graph convention f is defined on $(-1,1)$ and a limit can be taken $\lim_{x \to 0} f(x) = 2$

2.2.23 $f(x) = \dfrac{x^2 - 25}{x - 5}$; $a = 5$

Graph sketch: f undefined at $a = 5$ in given form



Rewrite f as

$f(x) = \dfrac{(x-5)(x+5)}{x-5} = x+5$ for $x \neq 5$

a straight line

$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = 10$ Since both one-sided limits exist so does $\lim_{x \to 5} f(x) = 10$.

2.2.33 a) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ does not exist

False. Rewrite $\dfrac{x^2 - 9}{x - 3} = \dfrac{(x-3)(x+3)}{x-3} = x + 3$ for $x \neq 3$ hence

$\lim_{x \to 3} = 6$

b) The value of $\lim_{x \to a} f(x)$ is always found by computing $f(a)$.

False. f can have a limit at $x \to a$, but not be defined at a, or

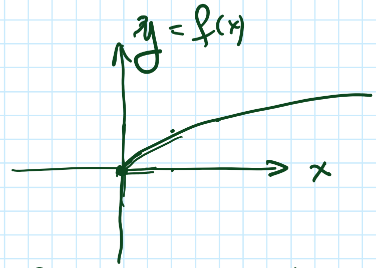
f can be discontinuous at $x=a$

c) The value of $\lim_{x\to a} f(x)$ does not exist if $f(a)$ is undefined.

False. For example $f(x)=\dfrac{x^2-9}{x-3}$ is not defined at $x=3$

but $\lim_{x\to 3} f(x) = 6$

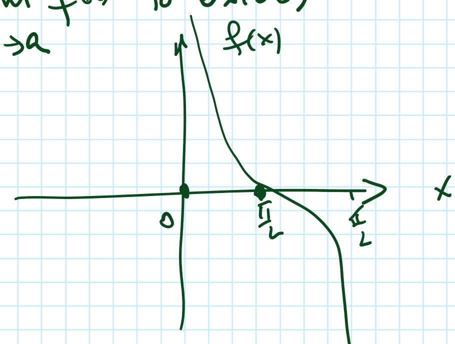
d) $\lim_{x\to 0} \sqrt{x} = 0$



False. $f(x)=\sqrt{x}$ is not defined for $x<0$ hence $\lim_{x\to 0^-} f(x)$ does not exist

(Theorem on one-sided limits states $\lim_{x\to a^\pm} f(x)$ must be equal for $\lim_{x\to a} f(x)$ to exist)

e) $\lim_{x\to \frac{\pi}{2}} \cot x = 0$

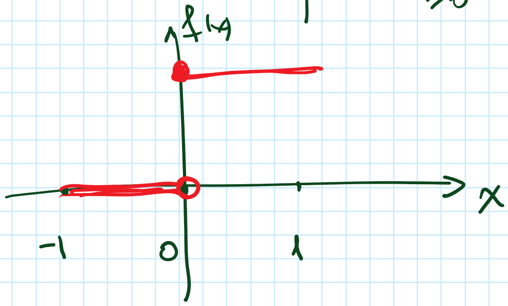


$f(x)=\cot x = \dfrac{\cos x}{\sin x}$

True. Cotangent is defined and has limit at $x=0$ (i.e., it is continuous.)

2.3.34. $H(x)=\begin{cases} 0 & x<0 \\ 1 & x\geq 0 \end{cases}$

a)



b) Since $\lim_{x\to 0^-} H(x)=0$ is different from $\lim_{x\to 0^+} H(x)=1$

the limit $\lim_{x\to 0} H(x)$ does not exist.

2.3.36 $\lim (1+2h)^{1/h}$ ...

2.3.36 $\lim\limits_{h\to 0} \dfrac{(1+2h)^{1/h}}{2e^{2+h}} \cong 0.5$

https://www.wolframalpha.com/input?i=plot+%281%2B2h%29%5E%281%2

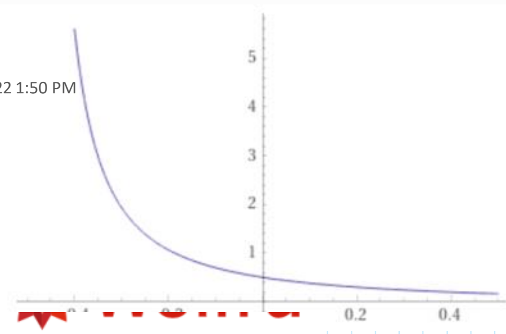
plot (1+2h)^(1/h)/2/e^(2+h) h from -1/2 to 1/2

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD

Input interpretation

plot	$\dfrac{\frac{1}{2}\sqrt[h]{1+2h}}{e^{2+h}}$	$h = -\frac{1}{2}$ to $\frac{1}{2}$

Plot



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Limit[(1/2 (1 + 2 h)^(1/h))/e^(2 + h), h->0]

NATURAL LANGUAGE | MATH INPUT

Limit

$$\lim_{h\to 0} \frac{\sqrt[h]{1+2h}}{2e^{2+h}} = \frac{1}{2}$$

Plot

2.3.38 $\lim\limits_{x\to 1} \dfrac{18(\sqrt[3]{x}-1)}{x^3-1} \cong 2$

From plot

plot 18(x^(1/3)-1)/(x^3-1) x from 0 to 2

NATURAL LANGUAGE | MATH INPUT

Assuming "0" is a decimal number | Use "0 to 2" as referring

Assuming the principal root | Use the real-valued root instea

limit 18(x^(1/3)-1)/(x^3-1) x->1

limit 18(x^(1/3)-1)/(x^3-1) x->1

NATURAL LANGUAGE | MATH INPUT

Assuming the principal root | Use the real-

Limit

$$\lim_{x\to 1} \frac{18\left(\sqrt[3]{x}-1\right)}{x^3-1} = 2$$

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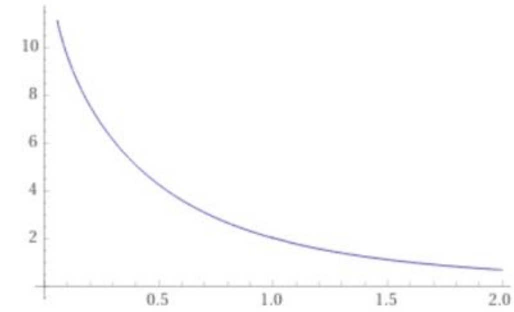
Assuming "0" is a decimal number | Use "0 to 2" as referring

Assuming the principal root | Use the real-valued root instea

Input interpretation

plot	$18\times\frac{\sqrt[3]{x}-1}{x^3-1}$	$x = 0$ to 2

Plot



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2.3.39. $\lim_{x\to 1} \frac{9\left(\sqrt{x-x^4} - \sqrt[3]{x}\right)}{1-x^{3/4}} = 16$ from plot and analytical evaluation

limit 9((2x-x^4)^(1/2)-x^(1/3))/(1-x^(3/4)) x->1

NATURAL LANGUAGE | MATH INPUT

Assuming the principal root | Use the real-valued root

Limit

$$\lim_{x\to 1} \frac{9\left(\sqrt{2x-x^4} - \sqrt[3]{x}\right)}{1-x^{3/4}} = 16$$

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Plot 9((2x-x^4)^(1/2)-x^(1/3))/(1-x^(3/4)) x from 0.9 to 1.1

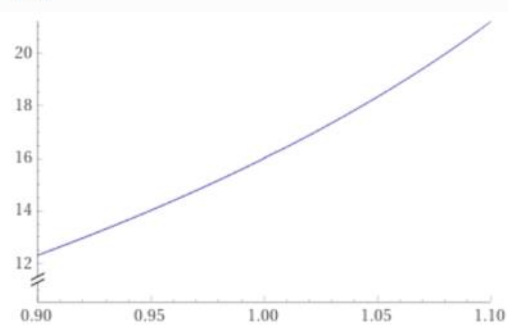
NATURAL LANGUAGE | MATH INPUT | EXTEN

Assuming the principal root | Use the real-valued root instead

Input interpretation

plot	$9\times\frac{\sqrt{2x-x^4}-\sqrt[3]{x}}{1-x^{3/4}}$	$x = 0.9$ to 1.1

Plot



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2.3.41 $\lim_{h\to 0} \frac{\ln(1+h)}{h} = 1$ from plot

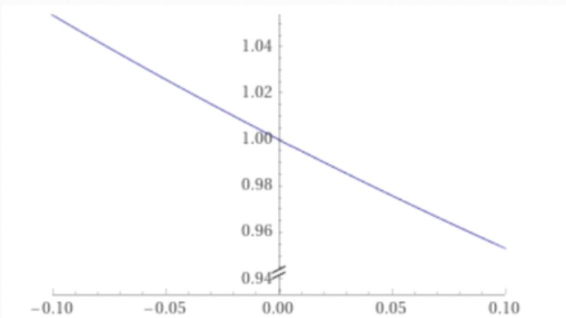
Plot ln(1+h)/h h from -0.1 to 0.1

NATURAL LANGUAGE MATH INPUT

Input interpretation

plot	$\frac{\log(1+h)}{h}$	$h = -0.1$ to 0.1

Plot



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limit ln(1+h)/h h->0

NATURAL LANGUAGE MATH

Limit

$$\lim_{h\to 0} \frac{\log(1+h)}{h} = 1$$

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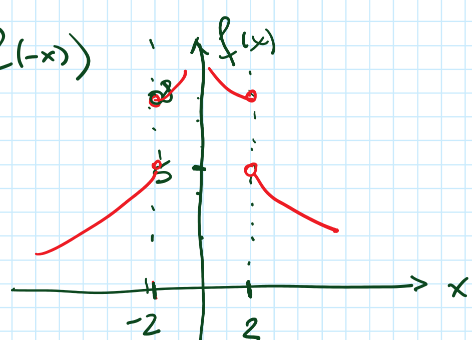
2.3.53 $f: \mathbb{R} \to \mathbb{R}$ even $(f(x) = f(-x))$

$\lim_{x\to 2^+} f(x) = 5$ $\quad \lim_{x\to 2^-} f(x) = 8$

Sketch a plot.



f even means the graph for $x<0$ is the mirror image of that for $x>0$

hence $\lim_{x\to -2^+} f(x) = 8$ $\quad \lim_{x\to -2^-} f(x) = 5$