HW02 Solution

Tuesday, September 6, 2022 2:11 PM

2.4.26 a) $\lim\limits_{x \to -2^+} \dfrac{x-4}{x(x+2)}$

For $x > -2$ close to -2 we have

$$\left.\begin{aligned} x-4 &\cong -6 < 0 \\ x &\cong -2 < 0 \\ x+2 &> 0 \end{aligned}\right| \Rightarrow \frac{x-4}{x(x+2)} > 0$$

The limit is positive infinity

$$\lim_{x \to -2^+} \frac{x-4}{x(x+2)} = \infty$$

b) $\lim\limits_{x \to -2^-} \dfrac{x-4}{x(x+2)} = -\infty$. As above but now $x+2 < 0$

c) $\lim\limits_{x \to -2} \dfrac{x-4}{x(x+2)}$ does not exist (different one-sided limits)

2.4.27 a) $\lim\limits_{x \to 2^+} \dfrac{x^2-4x+3}{(x-2)^2} = -\infty$ (Negative numerator divided by positive denominator with 0 limit)

For $x=2$, $x^2-4x+3 = 4-8+3 = -1 < 0$

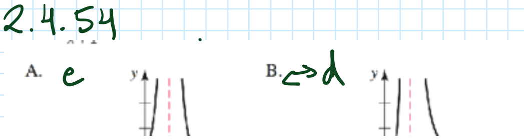
b) $\lim\limits_{x \to 2^-} \dfrac{x^2-4x+3}{(x-2)^2} = -\infty$. As above

c) $\lim\limits_{x \to 2} \dfrac{x^2-4x+3}{(x-2)^2} = -\infty$ (since both one-sided limits exist and are equal)

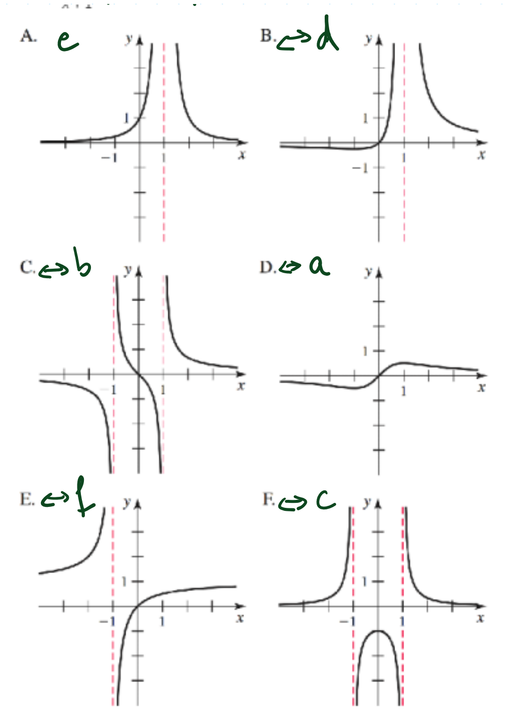
2.4.34 $\lim\limits_{t \to 5} \dfrac{4t^2-100}{t-5} \underset{\substack{\downarrow \\ \text{factor using} \\ (a^2-b^2)=(a-b)(a+b)}}{=} \lim\limits_{t \to 5} \dfrac{(2t-10)(2t+10)}{t-5} \underset{\substack{\downarrow \\ \text{(limit of product rule)}}}{=} 4 \lim\limits_{t \to 5} \dfrac{(t-5)(t+5)}{t-5} = 40$

2.4.35. $\lim\limits_{x \to 1^+} \dfrac{x^2-5x+6}{x-1} \underset{\substack{\downarrow \\ \text{factor}}}{=} \lim\limits_{x \to 1^+} \dfrac{\overbrace{(x-2)}^{\ominus}\overbrace{(x-3)}^{\ominus}}{\underbrace{x-1}_{\oplus}} = \infty$ (Positive denominator $\to 0$, Positive numerator)

2.4.54



a) $f(x) = \dfrac{x}{x^2+1}$



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a) $f(x) = \frac{x}{x^2+1}$

Since $f(0)=0$ and $\lim_{x\to\pm\infty} f(x) = 0$

(a) $\Leftrightarrow$ D

b) $f(x) = \frac{x}{x^2-1}$

$f(0)=0$; $\lim_{x\to\pm\infty} f(x) = 0$; $f(x)$ has vertical asymptotes at $x=\pm 1$

$\Rightarrow$ b $\Leftrightarrow$ C

c) $f(x) = \frac{1}{x^2-1}$

$f(0) = -1$; $\lim_{x\to\pm\infty} f(x) = 0$;

f has vert. asymp. at $x=\pm 1 \Rightarrow$

c $\Leftrightarrow$ F

d) $f(x) = \frac{x}{(x-1)^2}$

$f(0)=0$; $\lim f(x)=0$;

Vert. asymp. at $x=1 \Rightarrow$ d $\Leftrightarrow$ B

e) $f(x) = \frac{1}{(x-1)^2}$

$f(0)=1$ $\lim_{x\to\pm\infty} f(x) = 0$

Vert. Asymp at $x=1$

$\Rightarrow$ e $\Leftrightarrow$ A

f) $f(x) = \frac{x}{x+1}$ $f(0)=0$

$\lim_{x\to\pm\infty} f(x) = 1$; Vert. Asymp. at $x=-1$

$\Rightarrow$ f $\Leftrightarrow$ E.

2.5.37 $f(x) = \frac{4x}{20x+1}$ $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{4}{20+\frac{1}{x}} = \frac{1}{5}$

$\lim_{x\to-\infty} f(x) = \frac{1}{5}$ (as above). Vert. Asymptote at $20x+1=0 \Rightarrow x = -\frac{1}{20}$

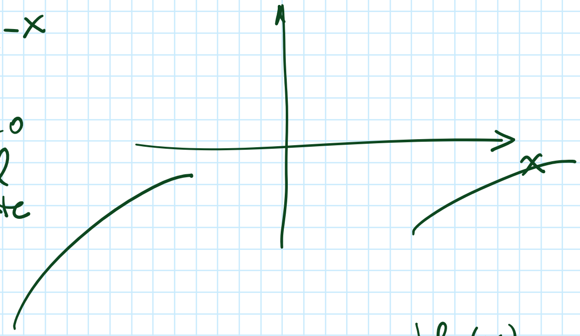
2.5.39 $f(x) = \frac{6x^2-9x+8}{3x^2+2}$

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{6-\frac{9}{x}+\frac{8}{x^2}}{3+\frac{2}{x^2}} = 2$$

No vertical asymptote since $3x^2+2>0$

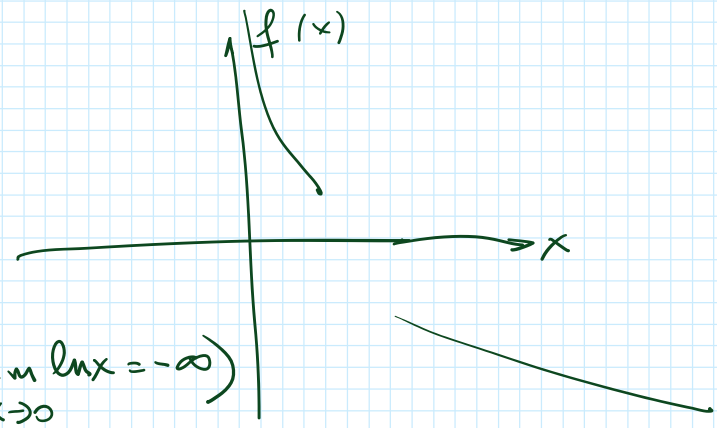
2.5.57 $f(x) = -3e^{-x}$

$\lim\limits_{x\to\infty} f(x) = 0$ with $y=0$ horizontal asymptote



$\lim\limits_{x\to-\infty} f(x) = -\infty$

2.5.59 $f(x) = 1 - \ln x$



$\lim\limits_{x\to\infty} f(x) = -\infty$

$\lim\limits_{x\to 0} f(x) = \infty$ (Since $\lim\limits_{x\to 0} \ln x = -\infty$)

2.5.71. $f(x) = \dfrac{2x^3+10x^2+12x}{x^3+2x^2}$

a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{2+\frac{10}{x}+\frac{12}{x^2}}{1+\frac{2}{x}} = 2$

$\lim\limits_{x\to-\infty} f(x) = 2$ (as above)

$y=2$ is a horizontal asymptote

b) $f(x)$ has a vertical asymptote at $x=0$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \dfrac{2x(x^2+5x+6)}{x^2(x+2)} = \lim\limits_{x\to 0^-} \dfrac{2x(x+2)(x+3)}{x^2(x+2)}$

$= \lim\limits_{x\to 0^-} \dfrac{2(x+3)}{x} = -\infty$

$\lim\limits_{x\to 0^+} f(x) = \infty$ (as above)