HW03 Solution

3.2.25 Compute derivative of $f(x) = \frac{1}{x+1}$ at $a = -\frac{1}{2}$; 5 through limits

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$ (State definition)

$$f'(a) = \lim_{h\to 0} \frac{\frac{1}{a+h+1} - \frac{1}{a+1}}{h}$$ (Replace definition of f)

$$f'(a) = \lim_{h\to 0} \frac{a+1 - (a+h+1)}{h(a+1)(a+h+1)}$$ Carry out algebraic calculations

$$f'(a) = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(a+1)(a+h+1)}$$

$$f'(a) = -\lim_{h\to 0} \frac{1}{(a+1)(a+h+1)} = -\frac{1}{(a+1)^2}$$

$$f'\left(-\tfrac{1}{2}\right) = -\frac{1}{\left(-\frac{1}{2}+1\right)^2} = -\frac{1}{\frac{1}{4}} = -4$$ Compute requested values

$$f'(5) = -\frac{1}{(5+1)^2} = -\frac{1}{36}$$

3.2.29. $f(s) = 4s^3 + 3s$; $a = -3, -1$

As above in 3.2.25

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{4(a+h)^3 + 3(a+h) - [4a^3 - 3a]}{h}$$

$$= \lim_{h\to 0} \frac{4[(a+h)^3 - a^3] + 3[(a+h) - a]}{h} =$$

$$\lim \quad 4[(a+h-a)((a+h)^2 + a(a+h) + a^2)] + 3[(a+h) - a]$$

$$= \lim_{h \to 0} \frac{\qquad\qquad\qquad}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[4\left((a+h)^2 + a(a+h) + a^2\right) + 3\right]}{\cancel{h}}$$

$$= 12a^2 + 3$$

$f'(-3) = 12(-3)^2 + 3 = 111 \qquad f'(-1) = 15.$

3.2.36 $f(x) = 5x^2 - 6x + 1 \qquad a = 2$

a) As above

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{5\left[(a+h)^2 - a^2\right] - 6\left[a+h-a\right]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[5\left((a+h)+a\right) - 6\right]}{\cancel{h}} = 10a - 6$$

$f'(2) = 14$

b) Find equation of line tangent to graph at $(a, f(a))$

Equation for line $y = mx + n$ (State line eq.)

Line passes through point $(a, f(a)) \Rightarrow f(a) = ma + n \Rightarrow$ (Impose cond.)

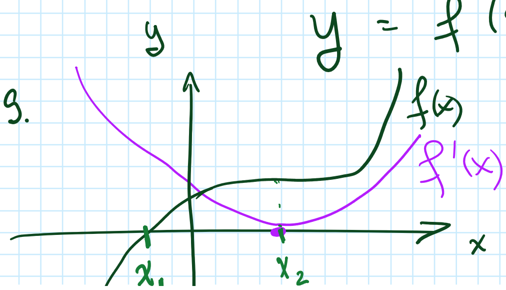
$$n = f(a) - ma$$

Line has slope $f'(a) \Rightarrow m = f'(a)$

Line equation $y = f'(a)x + f(a) - f'(a) \cdot a =$

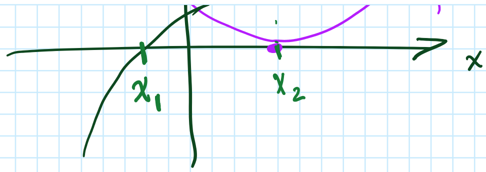
$$y = f'(a)(x-a) + f(a)$$

3.2.49.



x_1, x_2 Identify points of interest

x	$-\infty$		x_1		x_2		∞
f	$-\infty$	$\nearrow$	0	$\nearrow$		$\nearrow$	∞
f'	$+$		$+$		0	$+$	$+$



f	$-\infty$ ↗	0 ↗		↗	∞
f'	+	+	0	+	+

3.2.59. $f(x) = \begin{cases} 2x^2 & x \le 1 \\ ax-2 & x > 1 \end{cases}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \begin{cases} 4x & x \le 1 \\ a & x > 1 \end{cases}$$

Calculate limits.

$$\lim_{x \to 1_-} f'(x) = 4 \qquad \lim_{x \to 1_+} f'(x) = a$$

State theory

$f'(x)$ continuous if above one-sided limits are equal, hence $a=4$.

3.3.27. $p(x) = 8x \qquad p'(x) = 8 \qquad \left(\frac{d}{dx}(cf) = c\frac{df}{dx} \text{ rule} \right)$

3.3.28 $g(t) = 6\sqrt{t} = 6\, t^{\frac{1}{2}}$

$$g'(t) = 6\,\tfrac{1}{2}\, t^{\frac{1}{2}-1} = 3\,\frac{1}{t^{1/2}} = \frac{3}{\sqrt{t}}$$

3.3.33 $f(x) = 10x^4 - 32x + e^2 \qquad (e = \text{constant})$

$f'(x) = 40x^3 - 32 \qquad$ (diff. of polynomial)

3.3.68 $f(x) = 3x^3 + 5x^2 + 6x$

$$f'(x) = 9x^2 + 10x + 6$$

$$f''(x) = 18x + 10$$

$$f'''(x) = 18$$

3.3.71 $f(x) = \dfrac{x^2 - 7x - 8}{x+1}$

(Avoid lengthy calculation of derivative of quotient)

$$f(x) = \frac{(x-8)(x+1)}{x+1} = x-8$$

$$f'(x) = 1$$

$$f''(x) = 0$$

$$f'''(x) = 0$$