HW4 Solution

Thursday, September 22, 2022 1:05 PM

3.4.20 $g(x) = 6x - 2xe^x$

Sum rule $g(x) = u(x) - v(x)$ $u(x) = 6x$ $v(x) = 2xe^x$

$u'(x) = 6$

$g'(x) = u'(x) - v'(x) = 6 - v'(x)$

$v(x) = 2xe^x = a(x)\,b(x)$ $a(x) = 2x$, $a'(x) = 2$, $b(x) = e^x$, $b'(x) = e^x$

Product rule $v'(x) = a'(x)\,b(x) + a(x)\,b'(x)$

$v'(x) = 2e^x + 2xe^x = 2e^x(x+1)$

$g'(x) = 6 - 2e^x(x+1)$

Check in Wolfram Alpha

D[6x-2x Exp[x],x]

NATURAL LANGUAGE MATH INPUT

Derivative

$\frac{d}{dx}(6x - (2x)\exp(x)) = 6 - 2e^x(x+1)$

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3.4.22. $f(x) = \dfrac{x^3 - 4x^2 + x}{x-2} = \dfrac{g(x)}{h(x)}$

Quotient rule $f' = \dfrac{g'h - gh'}{h^2}$

$g(x) = x^3 - 4x^2 + x$ $g'(x) = 3x^2 - 8x + 1$

$h(x) = x - 2$ $h'(x) = 1$

$\Rightarrow$

D[(x^3-4x^2+x)/(x-2),x]

NATURAL LANGUAGE MATH INPUT

Derivative

$\frac{d}{dx}\left(\frac{x^3 - 4x^2 + x}{x - 2}\right) = \frac{2(x^3 - 5x^2 + 8x - 1)}{(x-2)^2}$

$f'(x) = \dfrac{(3x^2 - 8x + 1)(x-2) - (x^3 - 4x^2 + x)\cdot 1}{(x-2)^2} = \dfrac{3x^3 - 8x^2 + x - 6x^2 + 16x - 2 - x^3 + 4x^2 - x}{(x-2)^2} \Rightarrow$

$f'(x) = \dfrac{2x^3 - 10x^2 + 16x - 2}{(x-2)^2} = \dfrac{2(x^3 - 5x^2 + 8x - 1)}{(x-2)^2}$

e^x $g(x)$ $g(x) = e^x$ $g'(x) = e^x$

$$\frac{\quad}{e^x+1} = \frac{g}{h(x)} \qquad h(x) = e^x+1 \qquad h'(x) = e^x$$

$$f' = \frac{g'h - gh'}{h^2} = \frac{e^x(e^x+1) - e^x e^x}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2}$$

3.4.46 $h(x) = \frac{(x-1)(2x^2-1)}{(x^3-1)}$ h not defined at $x=1$

For $x \neq 1$ $\quad h(x) = \frac{(x-1)(2x^2-1)}{(x-1)(x^2+x+1)} = \frac{2x^2-1}{x^2+x+1} = \frac{f(x)}{g(x)}$

Quotient rule: $h' = \frac{f'g - fg'}{g^2}$ $\qquad f(x) = 2x^2-1 \quad f'(x) = 4x$

$g(x) = x^2+x+1 \quad g'(x) = 2x+1$

$$h'(x) = \frac{4x(x^2+x+1) - (2x^2-1)(2x+1)}{(x^2+x+1)^2} = \frac{\cancel{4x^3} + 4x^2 + 4x - \cancel{4x^3} - 2x^2 + 2x + 1}{(x^2+x+1)^2}$$

$$h'(x) = \frac{2x^2+6x+1}{(x^2+x+1)^2}$$

D[(x-1)(2x^2-1)/(x^3-1),x]

NATURAL LANGUAGE MATH INPUT

Derivative

$$\frac{d}{dx}\left(\frac{(x-1)(2x^2-1)}{x^3-1}\right) = \frac{2x^2+6x+1}{(x^2+x+1)^2}$$

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3.4.55 $h(r) = \frac{2-r-\sqrt{r}}{r+1} = \frac{f(r)}{g(r)}$

$$h' = \frac{f'g - fg'}{g^2}$$

$f(r) = 2-r-\sqrt{r}$; $f'(r) = -1 - \frac{1}{2\sqrt{r}}$

$g(r) = r+1$; $g'(r) = 1$

$$\Rightarrow h'(r) = \frac{-\left(1+\frac{1}{2\sqrt{r}}\right)(r+1) - (2-r-\sqrt{r})}{(r+1)^2}$$

$$h'(r) = -\frac{(2\sqrt{r}+1)(r+1) + 2\sqrt{r}(2-r-\sqrt{r})}{2\sqrt{r}\,(r+1)^2}$$

$$h'(r) = -\frac{\cancel{2r\sqrt{r}} + r + 2\sqrt{r} + 1 + 4\sqrt{r} - \cancel{2r\sqrt{r}} - 2r}{2\sqrt{r}\,(r+1)^2}$$

$$h'(r) = -\frac{1+6\sqrt{r}-r}{2\sqrt{r}\,(r+1)^2}$$

3.5.26 $\quad y(x) = \sin x + 4e^x \qquad y'(x) = \cos x + 4e^x$

3.5.29 $\quad y(x) = \dfrac{\cos x}{\sin x + 1} = \dfrac{f(x)}{g(x)}\ ; \quad y' = \dfrac{f'g - fg'}{g^2}$

$f(x) = \cos x \quad f'(x) = -\sin x\ ; \quad g(x) = \sin x + 1\ ; \quad g'(x) = \cos x$

$$y' = \frac{(-\sin x)(\sin x + 1) - \cos x \cdot \cos x}{(1+\sin x)^2} = \frac{-\sin^2 x - \cos^2 x - \sin x}{(1+\sin x)^2} =$$

$$= \frac{-1 - \sin x}{(1+\sin x)^2} = -\frac{1}{1+\sin x}.$$

3.5.40. $\quad y(x) = \dfrac{\sin x + \cos x}{e^x}$

$y = e^{-x}(\sin x + \cos x) = f \cdot g \qquad f(x) = e^{-x} \quad g(x) = \sin x + \cos x$

$y' = f'g + fg' \qquad f'(x) = -e^{x} \quad g'(x) = \cos x - \sin x$

$y'(x) = -e^{x}(\sin x + \cos x) + e^{-x}(\cos x - \sin x)$

$= -2e^{-x}\sin x$

3.5.48 $\quad y(t) = \dfrac{\tan t}{1 + \sec t} = \dfrac{\frac{\sin t}{\cos t}}{1 + \frac{1}{\cos t}} = \dfrac{\sin t}{1 + \cos t}$

$y = \dfrac{f}{g} \qquad y' = \dfrac{f'g - fg'}{g^2} \qquad f(t) = \sin t\ ; \quad f'(t) = \cos t$

$g(t) = 1 + \cos t\ ; \quad g'(t) = -\sin t$

$$y' = \frac{\cos t\,(1+\cos t) + \sin^2 t}{(1+\cos t)^2} = \frac{\cos t + 1}{(1+\cos t)^2} = \frac{1}{1+\cos t}$$

3.5.59 $\quad y(x) = e^x \sin x$

$y'(x) = e^x(\sin x + \cos x)$

$y'(x) = e^x(\cos x - \sin x) + e^x(\sin x + \cos x)$

$= 2e^x \cos x$

D[Exp[x] Sin[x],{x,2}]

NATURAL LANGUAGE MATH INPUT

Derivative

$$\frac{d^2}{dx^2}(\exp(x)\sin(x)) = 2\, e^x \cos(x)$$

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