

HW4 Solution

Thursday, September 22, 2022 1:05 PM

3.4.20 $g(x) = 6x - 2xe^x$

Sum rule $g(x) = u(x) - v(x)$ $u(x) = 6x$ $v(x) = 2xe^x$
 $u'(x) = 6$

$g'(x) = u'(x) - v'(x) = 6 - v'(x)$

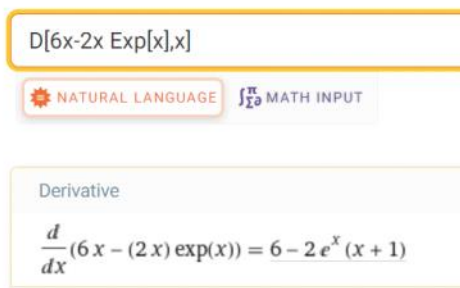
$v(x) = 2xe^x = a(x)b(x)$ $a(x) = 2x$, $a'(x) = 2$, $b(x) = e^x$, $b'(x) = e^x$

Product rule $v'(x) = a'(x)b(x) + a(x)b'(x)$

$v'(x) = 2e^x + 2xe^x = 2e^x(x+1)$

$g'(x) = 6 - 2e^x(x+1)$

Check in Wolfram Alpha



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3.4.22. $f(x) = \frac{x^3 - 4x^2 + x}{x-2} = \frac{g(x)}{h(x)}$

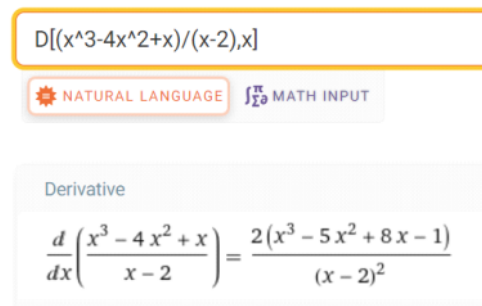
Quotient rule $f' = \frac{g'h - gh'}{h^2}$

$g(x) = x^3 - 4x^2 + x$ $g'(x) = 3x^2 - 8x + 1$
 $h(x) = x - 2$ $h'(x) = 1$

$f'(x) = \frac{(3x^2 - 8x + 1)(x-2) - (x^3 - 4x^2 + x) \cdot 1}{(x-2)^2} = \frac{3x^3 - 8x^2 + x - 6x^2 + 16x - 2 - x^3 + 4x^2 - x}{(x-2)^2} \Rightarrow$

$f'(x) = \frac{2x^3 - 10x^2 + 16x - 2}{(x-2)^2} = \frac{2(x^3 - 5x^2 + 8x - 1)}{(x-2)^2}$

e^x $g(x)$ $g(x) = e^x$ $g'(x) = e^x$



$$e^{x+1} = \frac{e}{h(x)} \quad \cup \quad h(x) = e^{x+1} \quad \cup \quad h'(x) = e^x$$

$$f' = \frac{g'h - gh'}{h^2} = \frac{e^x(e^{x+1}) - e^x e^x}{(e^{x+1})^2} = \frac{e^x}{(e^{x+1})^2}$$

3.4.46 $h(x) = \frac{(x-1)(2x^2-1)}{(x^3-1)}$ h not defined at $x=1$

For $x \neq 1$ $h(x) = \frac{(x-1)(2x^2-1)}{(x-1)(x^2+x+1)} = \frac{2x^2-1}{x^2+x+1} = \frac{f(x)}{g(x)}$

Quotient rule: $h' = \frac{f'g - fg'}{g^2}$ $f(x) = 2x^2 - 1$ $f'(x) = 4x$
 $g(x) = x^2 + x + 1$ $g'(x) = 2x + 1$

$$h'(x) = \frac{4x(x^2+x+1) - (2x^2-1)(2x+1)}{(x^2+x+1)^2} = \frac{4x^3 + 4x^2 + 4x - 4x^3 - 2x^2 + 2x + 1}{(x^2+x+1)^2}$$

$$h'(x) = \frac{2x^2 + 6x + 1}{(x^2+x+1)^2}$$

D[(x-1)(2x^2-1)/(x^3-1),x]

NATURAL LANGUAGE MATH INPUT

Derivative

$$\frac{d}{dx} \left(\frac{(x-1)(2x^2-1)}{x^3-1} \right) = \frac{2x^2+6x+1}{(x^2+x+1)^2}$$

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3.4.55 $h(r) = \frac{2-r-\sqrt{r}}{r+1} = \frac{f(r)}{g(r)}$

$$h' = \frac{f'g - fg'}{g^2}$$

$$\left. \begin{array}{l} f(r) = 2-r-\sqrt{r}; \quad f'(r) = -1 - \frac{1}{2\sqrt{r}} \\ g(r) = r+1; \quad g'(r) = 1 \end{array} \right\} \Rightarrow h'(r) = \frac{-\left(1 + \frac{1}{2\sqrt{r}}\right)(r+1) - (2-r-\sqrt{r})}{(r+1)^2}$$

$$h'(r) = - \frac{(2\sqrt{r}+1)(r+1) + 2\sqrt{r}(2-r-\sqrt{r})}{2\sqrt{r}(r+1)^2}$$

$$h'(r) = - \frac{2r\sqrt{r} + r + 2\sqrt{r} + 1 + 4\sqrt{r} - 2r\sqrt{r} - 2r}{2\sqrt{r}(r+1)^2}$$

$$h'(r) = - \frac{1 + 6\sqrt{r} - r}{2\sqrt{r}(r+1)^2}$$

$$3.5.26 \quad y(x) = \sin x + 4e^x \quad y'(x) = \cos x + 4e^x$$

$$3.5.29 \quad y(x) = \frac{\cos x}{\sin x + 1} = \frac{f(x)}{g(x)}; \quad y' = \frac{f'g - fg'}{g^2}$$

$$f(x) = \cos x \quad f'(x) = -\sin x; \quad g(x) = \sin x + 1; \quad g'(x) = \cos x$$

$$y' = \frac{(-\sin x)(\sin x + 1) - \cos x \cdot \cos x}{(1 + \sin x)^2} = \frac{-\sin^2 x - \cos^2 x - \sin x}{(1 + \sin x)^2} =$$

$$= \frac{-1 - \sin x}{(1 + \sin x)^2} = -\frac{1}{1 + \sin x}$$

$$3.5.40. \quad y(x) = \frac{\sin x + \cos x}{e^x}$$

$$y = e^{-x} (\sin x + \cos x) = f \cdot g \quad f(x) = e^{-x} \quad g(x) = \sin x + \cos x$$

$$y' = f'g + fg'$$

$$f'(x) = -e^{-x} \quad g'(x) = \cos x - \sin x$$

$$y'(x) = -e^{-x} (\sin x + \cos x) + e^{-x} (\cos x - \sin x)$$

$$= -2e^{-x} \sin x$$

$$3.5.48 \quad y(t) = \frac{\tan t}{1 + \sec t} = \frac{\frac{\sin t}{\cos t}}{1 + \frac{1}{\cos t}} = \frac{\sin t}{1 + \cos t}$$

$$y = \frac{f}{g} \quad y' = \frac{f'g - fg'}{g^2}$$

$$f(t) = \sin t; \quad f'(t) = \cos t$$

$$g(t) = 1 + \cos t; \quad g'(t) = -\sin t$$

$$y' = \frac{\cos t (1 + \cos t) + \sin t}{(1 + \cos t)^2} = \frac{\cos t + 1}{(1 + \cos t)^2} = \frac{1}{1 + \cos t}$$

3.5.59

$$y(x) = e^x \sin x$$

$$y'(x) = e^x (\sin x + \cos x)$$

$$y'(x) = e^x (\cos x - \sin x) + e^x (\sin x + \cos x)$$

$$= 2e^x \cos x$$

D[Exp[x] Sin[x]},{x,2}]

 NATURAL LANGUAGE  MATH INPUT

Derivative

$$\frac{d^2}{dx^2}(\exp(x) \sin(x)) = 2 e^x \cos(x)$$

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