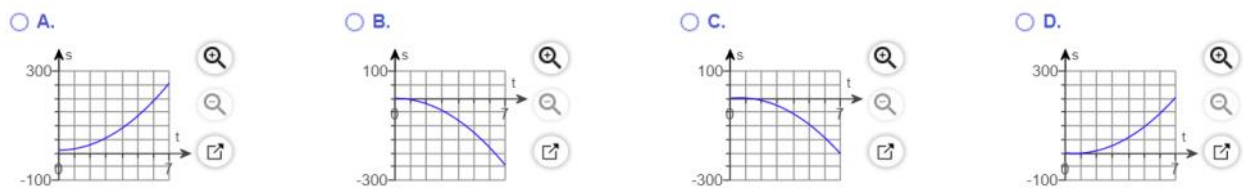
HW05 solution

Saturday, October 1, 2022 9:15 AM

3.6.15 $s = f(t)$ $\quad f(t) = 5t^2 - 6t \quad 0 \le t \le 7$

a. Graph the position function. Choose the correct graph below.



Avoid simply choosing an option. Rather, form the table to draw a qualitative plot

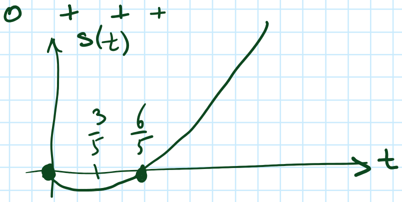
t	0		$\frac{3}{5}$		$\frac{6}{5}$	
$s(t)$	0	↘	$-\frac{6}{5}$	↗	0	↗
$s'(t)$	$-$	$-$ $-$	0	$+$	$+$	$+$

$S(t) = 5t^2 - 6t = t(5t-6)$

$S(0) = 0$; $S\left(\frac{6}{5}\right) = 0$; $S(7) = 7 \cdot 29 = 203$

$S'(t) = 10t - 6$; $S'\left(\frac{3}{5}\right) = 0$; $S\left(\frac{3}{5}\right) = \frac{3}{5}(-2) = -\frac{6}{5}$

$S'(0) = -6$

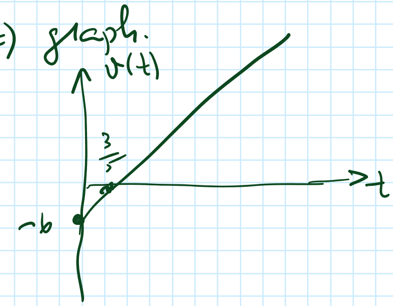


D is only choice consistent with $s(t)$ graph.

b. $v(t) = s'(t)$ (definition of velocity)

$v(t) = 10t - 6 \qquad v(0) = -6 \qquad v(7) = 64$

$v\left(\frac{3}{5}\right) = 0$



stationary $\Rightarrow v(t) = 0$; at $t_1 = \frac{3}{5}$

moving to left $t < \frac{3}{5}$; moving to right $t > \frac{3}{5}$

c. $a(t) = v'(t)$ (definition of acceleration) $\quad a(t) = 10$

$v(1) = 4 \qquad a(1) = 10$

d. $a(t) = \text{constant} = 10$

e. $a(t) > 0 \Rightarrow$ velocity is always increasing for $0 \le t \le 7$

3.6.18. $s(t) = 18t - 3t^2 \quad 0 \le t \le 8$

As above, construct table & sketch function plot

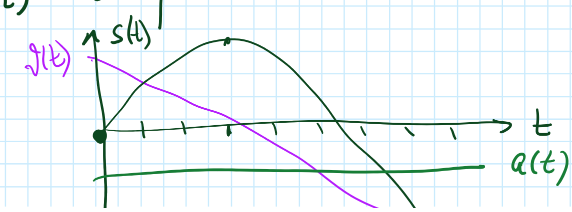
t	0		3		8
$s(t) = 3t(6-t)$	0	↗	27	↘	-48
$v(t) = s'(t) = 6(3-t)$	$+$	$+$ $+$	0	$-$ $-$	$-$
$a(t) = s''(t) = -6$	$-$	$-$ $-$	$-$	$-$ $-$	$-$

a. A consistent with sketch

b. $v(t) = 6(3-t)$

c. $v(1) = 12$; $a(1) = -6$

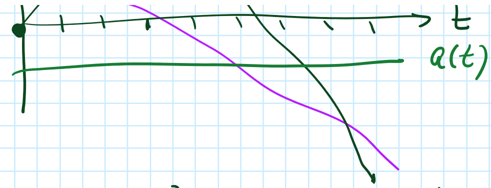
d. $a(t) = \text{constant} = -6$



c. $v(1) = 12$; $a(1) = -6$

d. $a(t) = \text{constant} = -6$

e. $a < -6$; velocity always decreasing

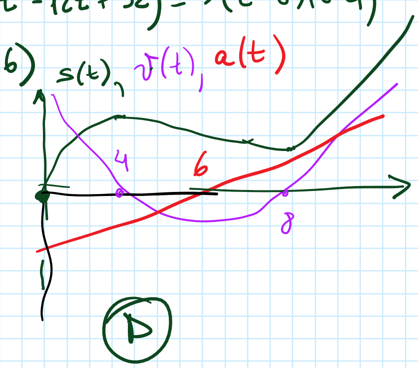


3.6.19 As above: distance $s(t) = t^3 - 18t^2 + 96t = t(t^2 - 18t + 96)$ $\quad 0 \le t \le 10$

velocity $v(t) = s'(t) = 3t^2 - 36t + 96 = 3(t^2 - 12t + 32) = 3(t-8)(t-4)$

acceleration $a(t) = v'(t) = 6t - 36 = 6(t-6)$

t	0			4			6			8		10
s(t)	0	↗			↘		↘					↗
v(t)	96	+	+	0	−	−	−			0	+	+ +
a(t)	−36	−	−	−			0	+	+	+		+60



$S(4) = 4(16 - 72 + 96) = 160$

$S(8) = 4(64 - 144 + 96) = 64$

3.6.37 Again, avoid simply picking a pull-down menu item. Work out the sol.

A stone is thrown vertically into the air at an initial velocity of 99 ft/s. On a different planet, the height s (in feet) of the stone above the ground after t seconds is $s = 99t - 5t^2$ and on Earth it is $s = 99t - 16t^2$. How much higher will the stone travel on the other planet than on Earth?

Screen clipping taken: 10/1/2022 10:23 AM

Planet X $\qquad s(t) = 99t - 5t^2$

Earth $\qquad h(t) = 99t - 16t^2$

At heighest point velocity is zero $\Rightarrow$ On X: $s'(t_X) = 0 \Rightarrow 99 - 10t_X = 0 \Rightarrow t_X = \frac{99}{10}$

On Earth: $h'(t_E) = 0 \Rightarrow 99 - 32t_E = 0 \Rightarrow t_E = \frac{99}{32}$

Highest points. On X: $s(t_X) = 99\frac{99}{10} - 5\frac{99^2}{10^2} = \frac{99^2}{10}\left(1 - \frac{5}{10}\right) = \frac{99^2}{20}$

On Earth $h(t_E) = 99\frac{99}{32} - 16\frac{99^2}{32^2} = \frac{99^2}{32}\left(1 - \frac{1}{2}\right) = \frac{99^2}{64}$

Difference in heights: $s(t_X) - h(t_E) = 99^2\left(\frac{1}{20} - \frac{1}{64}\right)$.

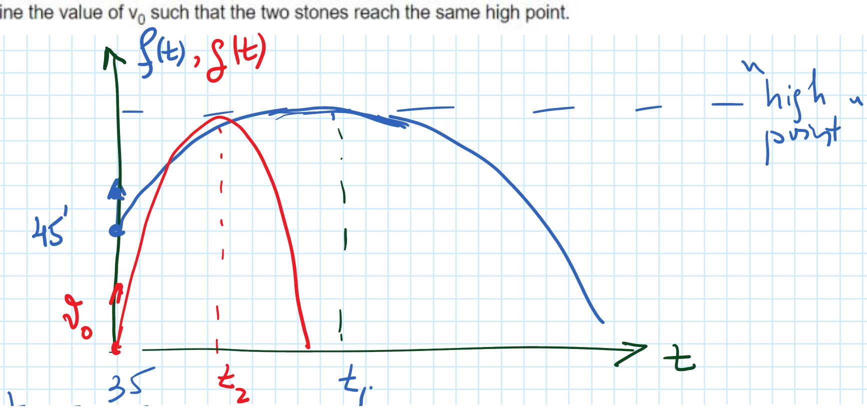
3.6.39

A stone is thrown with an initial velocity of 35 ft/s from the edge of a bridge that is 45 ft above the ground. The height of this stone above the ground t seconds after it is thrown is $f(t) = -16t^2 + 35t + 45$. If a second stone is thrown from the ground, then its height above the ground after t seconds is given by $g(t) = -16t^2 + v_0 t$, where v_0 is the initial velocity of the second stone. Determine the value of v_0 such that the two stones reach the same high point.

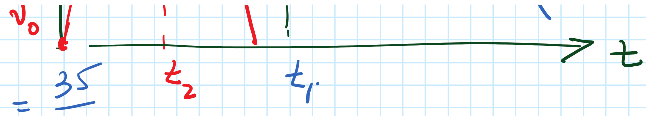
Screen clipping taken: 10/1/2022 10:32 AM

Draw a sketch as you read the problem

Stone 1 reaches high point when $v_1(t_1) = f'(t_1) = 0$



when $v_1(t_1) = f'(t_1) = 0$



$f'(t) = -32t + 35; \quad t_1 = \frac{35}{32}$

Stone 2 reaches high point when $v_2(t_2) = g'(t_2) = 0$

$g'(t) = -32t + v_0 \Rightarrow t_2 = \frac{v_0}{32}$

Stones must reach same "high point" $\Rightarrow f(t_1) = g(t_2)$

$\underbrace{f(t_1)}_{\text{(stone 1) height}} = \underbrace{g(t_2)}_{\text{(stone 2 height)}}$

$$\left.\begin{array}{l} f(t_1) = -16\left(\frac{35}{32}\right)^2 + 35\left(\frac{35}{32}\right) + 45 \\ g(t_2) = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) \end{array}\right\} \Rightarrow \frac{v_0^2}{32}\left(1 - \frac{1}{2}\right) = \frac{35^2}{32}\left(1 - \frac{1}{2}\right) + 45 \Rightarrow$$

$$\frac{v_0^2}{64} = \frac{35^2}{64} + 45 \Rightarrow v_0^2 = 35^2 + 45 \cdot 64 \Rightarrow v_0 \cong 64 \text{ ft/s}$$

3.7.17 $\quad y(x) = \sin^{13} x = f(g(x)) \qquad f(u) = u^{13} \quad g(x) = \sin x$

$y'(x) = 13 \sin^{12} x \cos x \qquad f'(u) = 13u^2 \quad g'(x) = \cos x$

3.7.20 $\quad y(x) = \sqrt{-1+7x} = f(g(x)) \qquad f(u) = \sqrt{u} \quad g(x) = 7x - 1$

$y'(x) = \frac{7}{2\sqrt{7x-1}} \qquad f'(u) = \frac{1}{2\sqrt{u}} \quad g'(x) = 7$

3.7.23 $\quad y(x) = \tan(4x^2) = f(g(x)) \qquad f(u) = \tan u \quad g(x) = 4x^2$

$f'(u) = \frac{1}{\cos^2 u} \quad g'(x) = 8x$

$y'(x) = \frac{8x}{\cos^2(4x^2)}$

3.7.44 $\quad y(x) = \cos(8 \sin x) \qquad f(u) = \cos u \quad g(x) = 8 \sin x$

$f'(u) = -\sin u \quad g'(x) = 8 \cos x$

$y'(x) = -[\sin(8 \sin x)] \cdot [8 \cos x] = -8 \cos x \sin(8 \sin x)$

3.7.49 $\quad y(x) = \sqrt{8 + \cot^2 x} \qquad f(u) = \sqrt{u} \quad g(x) = 8 + \cot^2 x$

$f'(u) = \frac{1}{2\sqrt{u}}; \quad g'(x) = \frac{d}{dx}(\cot^2 x)$

Apply diff. comp. functions for $\cot^2 x$ also:

$\cot^2 x = a(b(x)) \qquad a(u) = u^2 \quad b(x) = \cot x$

$a'(u) = 2u \quad b'(x) = -\csc^2 x$

$\frac{d}{\ }\ ^2$

$$\frac{d}{dx}\cot x = 2\cot x \cdot (-\csc^2 x) \Rightarrow$$

$$g'(x) = -2(\cot x)(\csc^2 x)$$

$$y'(x) = -\frac{(\cot x)(\csc^2 x)}{\sqrt{8+\cot^2 x}}$$