S06

Friday, October 7, 2022 11:22 AM

3.8.15 $y^2 = 4x$; $(1,2) = (x_0, y_0)$

$\frac{d}{dx}$: $2yy' = 4 \Rightarrow y' = \frac{2}{y} = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$; $y'(1) = 1$

3.8.17 $\sin y = 5x^4 - 5$; $(1, \pi) = (x_0, y_0)$

$\frac{d}{dx}$: $(\cos y) y' = 20x^3 \Rightarrow y' = \frac{20x^3}{\cos y} = \frac{20x^3}{\sqrt{1-\sin^2 y}} = \frac{20x^3}{\sqrt{1-25(x^4-1)^2}}$

At $(x_0, y_0) = (1, \pi)$ $y' = \frac{20 \cdot 1}{\cos \pi} = -20$

3.8.18. $\sqrt{x} - 2\sqrt{y} = 0$; $(4,1)$

$\frac{d}{dx}$: $\frac{1}{2\sqrt{x}} - 2\frac{1}{2\sqrt{y}} y' = 0 \Rightarrow y' = \frac{1}{2}\sqrt{\frac{y}{x}}$

At $(4,1)$ $y' = \frac{1}{2}\sqrt{\frac{4}{1}} = 1$

3.8.45 $\sin y + 5x = y^2$; $(0,0) = (x_0, y_0)$

Verify: $\sin 0 + 5 \cdot 0 = 0$ ✓

Tangent line: $y - y_0 = m \cdot (x - x_0)$ $m = y'(x_0)$

$\frac{d}{dx}$: $(\cos y) y' + 5 = 2yy' \Rightarrow y' = \frac{5}{2y - \cos y}$ $\Rightarrow$

At $(0,0)$ $y'(0) = \frac{5}{2 \cdot 0 - 1} = -5$

Tangent line $y = -5x$

3.8.46. $x^3 + y^3 = 2xy$; $(1,1) = (x_0, y_0)$

Verify (1,1) satisfies eq. : $1 + 1 = 2 \cdot 1 \cdot 1$ ✓

$\frac{d}{dx}$: $3(x^2 + y^2 y') = 2(y + xy') \Rightarrow$

$(3y^2 - 2x) y' = 2y - 3x^2 \Rightarrow y' = \frac{2y - 3x^2}{3y^2 - 2x}$

At $(1,1)$ $y'(1) = \frac{2-3}{3-2} = -1 = m$

Tangent line $y-1 = m(x-1) \Rightarrow y-1 = 1-x \Rightarrow$

$$x+y=2.$$

3.9.36. $y = \ln (x^3+1)^{\pi} = \pi \ln (x^3+1)$

Composite diff. rule $y(x) = f(x) g(x)$ $y'(x) = f'(u) g'(x)$ $u = g(x)$

Identify: outer function $f(u) = \ln u^{\pi} = \pi \ln u$ $f'(u) = \frac{\pi}{u}$

inner function $g(x) = x^3+1$; $g'(x) = 3x^2$

$$y'(x) = \frac{\pi}{x^3+1} \cdot 3x^2 = \frac{3\pi x^2}{x^3+1}$$

3.9.37 $y = 8^x$

This is of form $y = b^x$ $y' = \ln b \cdot b^x$, in this case $y' = \ln 8 \cdot 8^x$.

Can also use logarithmic differentiation:

$$\ln y = \ln 8^x = x \ln 8$$

$$\frac{d}{dx}: \quad \frac{1}{y} y' = \ln 8 \Rightarrow y' = (\ln 8) y = (\ln 8) \cdot 8^x.$$

3.9.40 $y = 4^{-x} \sin x$

Product rule $y(x) = f(x) g(x) \Rightarrow y' = f'g + fg'$ $\Big\} \Rightarrow$

$f = 4^{-x}$ $f' = -\ln 4 \, 4^{-x}$; $g = \sin x$ $g' = \cos x$

$$y' = (-\ln 4) 4^{-x} \sin x + 4^{-x} \cos x = (-\ln 4 \cdot \sin x + \cos x) 4^{-x}$$

logarithmic differentiation (alternative solution)

$$\ln y = \ln (4^{-x} \sin x) = -x \ln 4 + \ln \sin x$$

$$\frac{d}{dx}: \quad \frac{y'}{y} = -\ln 4 + \frac{\cos x}{\sin x} \Rightarrow$$

$$y' = \left(-\ln 4 + \frac{\cos x}{\sin x}\right) 4^{-x} \sin x = (-\ln 4 \cdot \sin x + \cos x) 4^{-x}. \checkmark$$

3.9.77 $f(x) = \frac{(x+10)^{10}}{(2x-4)^8}$

$\frac{d}{dx}$: $\ln f = 10 \ln(x+10) - 8 \ln(2x-4)$

$\frac{f'}{f} = \frac{10}{x+10} - \frac{8}{2x-4} = \frac{10}{x+10} - \frac{4}{x-2} \Rightarrow$

$f' = \left(\frac{10}{x+10} - \frac{4}{x-2} \right) \frac{(x+10)^{10}}{(2x-4)^8}$

3.978 $f(x) = x^2 \cos x$

$\ln f = 2 \ln x + \ln \cos x$

$\frac{d}{dx}$: $\frac{f'}{f} = \frac{2}{x} - \frac{\sin x}{\cos x} \Rightarrow f' = \left(\frac{2}{x} - \frac{\sin x}{\cos x} \right) x^2 \cos x$

$f'(x) = 2x \cos x - x^2 \sin x.$