HW 11 - Solution

Friday, November 11, 2022 10:40 AM

5.1.7, 9, 19 ⟶ see L23, solutions drafted in classtime.

5.1.15. Organize calculations in a table

$v(t) = 3t^2+1$ velocity

$v: [0,4] \to \mathbb{R}$

$x(t)$ displacement

$x'(t) = v(t)$

$x(0) = 0$; $\Delta x = 1$

$x(1) \cong x(0) + v(0.5)\cdot\Delta x$

$= 0 + 1.75 \cdot 1 = 1.75$

$x(2) \cong x(1) + v(1.5)\cdot \Delta x$

$= 1.75 + 7.75 = 9.50$

$x(3) = 29.25$

$x(4) = 67.00$

Table[3t^2+1, {t,0.5, 3.5}]

NATURAL LANGUAGE | MATH INPUT

Input

$\text{Table}[3t^2+1, \{t, 0.5, 3.5\}]$

Result

t	0.5	1.5	2.5	3.5
$3t^2+1$	1.75	7.75	19.75	37.75

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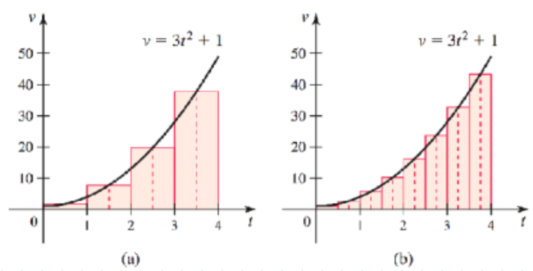
15. Approximating displacement The velocity in ft/s of an object moving along a line is given by $v = 3t^2+1$ on the interval $0 \le t \le 4$, where t is measured in seconds.

a. Divide the interval [0, 4] into $n = 4$ subintervals, [0, 1], [1, 2], [2, 3], and [3, 4]. On each subinterval, assume the object moves at a constant velocity equal to v evaluated at the midpoint of the subinterval and use these approximations to estimate the displacement of the object on [0, 4] (see part (a) of the figure).

b. Repeat part (a) for $n = 8$ subintervals (see part (b) of the figure).



(a) (b)

25–32. Left and right Riemann sums *Complete the following steps for the given function, interval, and value of n.*

a. *Sketch the graph of the function on the given interval.*

b. *Calculate Δx and the grid points $x_0, x_1, \ldots, x_n$.*

c. *Illustrate the left and right Riemann sums. Then determine which Riemann sum underestimates and which sum overestimates the area under the curve.*

d. *Calculate the left and right Riemann sums.*

25. $f(x) = x+1$ on $[0, 4]$; $n = 4$

26. $f(x) = 9-x$ on $[3, 8]$; $n = 5$

27. $f(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$; $n = 4$

28. $f(x) = \sin^{-1}\frac{x}{3}$ on $[0, 3]$; $n = 6$

29. $f(x) = x^2 - 1$ on $[2, 4]$; $n = 4$

30. $f(x) = 2x^2$ on $[1, 6]$; $n = 5$

31. $f(x) = e^{x/2}$ on $[1, 4]$; $n = 6$

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5.1.27

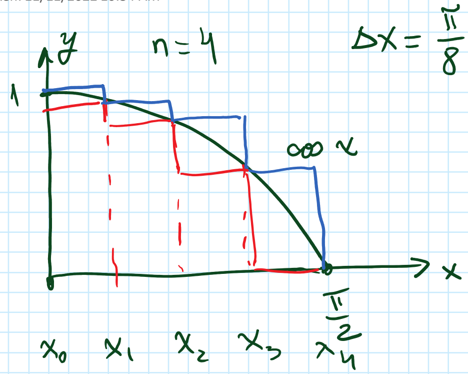
— Left sum

Overestimates

— Right sum

Underestimates

$n = 4$ $\Delta x = \frac{\pi}{8}$



Table[N[Cos[x]], {x,0,Pi/2,Pi/8}]

NATURAL LANGUAGE | MATH INPUT | EXTEND

Input

$\text{Table}[N[\cos(x)], \{x, 0, \frac{\pi}{2}, \frac{\pi}{8}\}]$

Result

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$\cos(x)$	1	0.92388	0.707107	0.382683	0

$$\text{Left sum} = \Delta x \sum_{k=0}^{n-1} \cos x_k =$$

$$\text{Left sum} = \Delta x \sum_{k=0}^{n-1} \cos x_k =$$
$$= \frac{\pi}{8} \sum_{k=0}^{3} \cos \frac{k\pi}{8}$$

$$\text{Right sum} = \Delta x \sum_{k=1}^{n} \cos x_k =$$
$$= \frac{\pi}{8} \sum_{k=1}^{4} \cos \frac{k\pi}{8}$$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$\cos(x)$	1	0.92388	0.707107	0.382683	0

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$$\sum_{k=0}^{3} \cos\left(k*\frac{Pi}{8}\right)$$

NATURAL LANGUAGE MATH INPUT

POPULAR

Sum

$$\sum_{k=0}^{3} \cos\left(\frac{\pi k}{8}\right) = 1 + \frac{1}{\sqrt{2}} + \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right)$$

Decimal approximation

3.0136697460629240522574875355320361928685

$$\sum_{k=1}^{4} \cos\left(k*\frac{Pi}{8}\right)$$

NATURAL LANGUAGE MATH INPUT

POPULAR

Sum

$$\sum_{k=1}^{4} \cos\left(\frac{\pi k}{8}\right) = \frac{1}{\sqrt{2}} + \sin\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{8}\right)$$

Decimal approximation

2.01366974606292405225748753553203

...

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5.1.31. As above, replace $\cos x$ on $[0,4]$ with $e^{x/2}$ on $[1,4]$

5.1.40. —||—

5.1.45.

45. Displacement from a table of velocities The velocities (in mi/hr) of an automobile moving along a straight highway over a two-hour period are given in the following table.

t (hr)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
v (mi/hr)	50	50	60	60	55	65	50	60	70

a. Sketch a smooth curve passing through the data points.

b. Find the midpoint Riemann sum approximation to the displacement on [0, 2] with $n = 2$ and $n = 4$.

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a) $v(t)$

70

b) $n=2 \Rightarrow \Delta t = 1$

$$\text{Mid. Sum} = \Delta t \left[v(0.5) + v(1.5) \right] =$$

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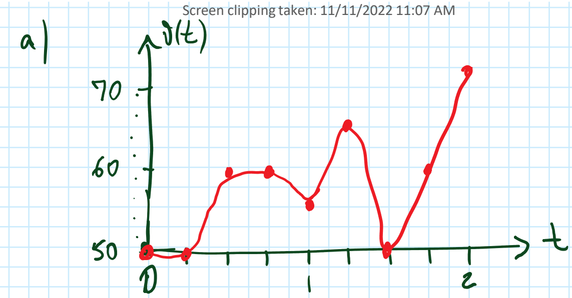
a)



b) $n=2 \Rightarrow \Delta t = 1$

Mid. Sum $= \Delta t\,[v(0.5) + v(1.5)] =$

$= 1\cdot[60 + 50] = 110$ mi

$n=4 \Rightarrow \Delta t = 0.5$

Mid Sum $= \Delta t\,[v(0.25) + v(0.75) + v(1.25) +$

$v(1.75)] = \dots$

5.1.49.

49. Sigma notation Evaluate the following expressions.

a. $\sum_{k=1}^{10} k$

b. $\sum_{k=1}^{6} (2k+1)$

c. $\sum_{k=1}^{4} k^2$

d. $\sum_{n=1}^{5} (1+n^2)$

e. $\sum_{m=1}^{3} \frac{2m+2}{3}$

f. $\sum_{j=1}^{3} (3j-4)$

g. $\sum_{p=1}^{5} (2p+p^2)$

h. $\sum_{n=0}^{4} \sin\frac{n\pi}{2}$

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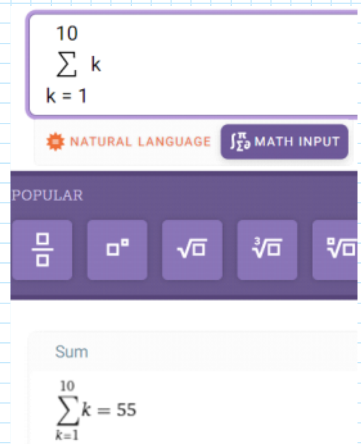
a) $\sum_{k=1}^{10} k = 1+2+\dots+10 = \frac{10(10+1)}{2} = 55$. Check in Wolfram Alpha



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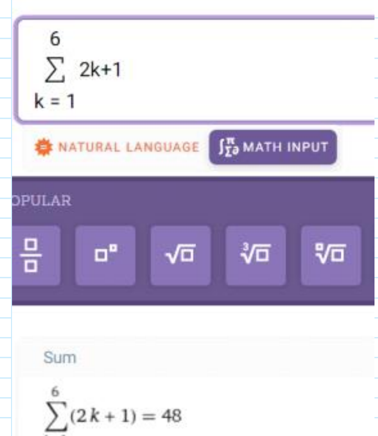
In general: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

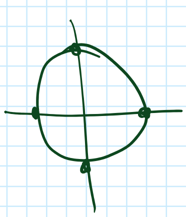
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Apply: $\sum_{k=1}^{6} (2k+1) = 2\sum_{k=1}^{6} k + \sum_{k=1}^{6} 1 = 2\,\frac{6(6+1)}{2} + 6 = 48$



h. $\sum_{n=0}^{4} \sin\frac{n\pi}{2} = \sin 0 + \sin\frac{\pi}{2} + \sin\pi + \sin\frac{3\pi}{2} + \sin 2\pi$

$= 0 + 1 + 0 - 1 + 0 = 0.$



5.1.64. **64.** The midpoint Riemann sum for $f(x) = 1 + \cos\pi x$ on $[0, 2]$ with $n = 50$

$S = \Delta x \sum_{k=1}^{n} \left[1 + \cos\frac{\pi}{2}(x_k + x_{k-1})\right] =$

$n = 50$

$\Delta x = \frac{2}{n} = \frac{2}{50} = \frac{1}{25} = 0.04$

$x_k = k\,\Delta x \qquad k = 0, 1, \dots, n$

$S = n\Delta x + \Delta x \sum_{k=1}^{n} \cos\left[\pi\left(k-\frac{1}{2}\right)\Delta x\right]$ ... Δx

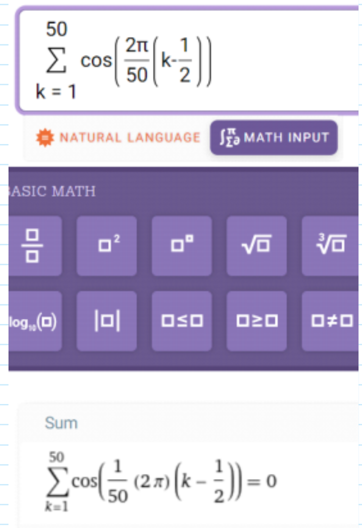
$k=1$

$$S = n\Delta x + \Delta x \sum_{k=1}^{n} \cos\left[\pi\left(k-\tfrac{1}{2}\right)\Delta x\right]$$

$$x_k = k\Delta x \qquad k=0,1,\dots,n$$

$$\frac{x_k + x_{k-1}}{2} = \frac{\Delta x}{2}(k+k-1) = k\Delta x - \frac{\Delta x}{2}$$
$$= \left(k-\tfrac{1}{2}\right)\Delta x$$

$$S = 2 + \frac{2}{n} \sum_{k=1}^{n} \cos\left[\frac{2\pi}{n}\left(k-\tfrac{1}{2}\right)\right]$$



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