HW 12 Solution

Saturday, November 19, 2022 7:29 PM

5.2.26.

**21–26. Approximating net area** *The following functions are positive and negative on the given interval.*

a. *Sketch the function on the interval.*

b. *Approximate the net area bounded by the graph of f and the x-axis on the interval using a left, right, and midpoint Riemann sum with $n = 4$.*

c. *Use the sketch in part (a) to show which intervals of $[a, b]$ make positive and negative contributions to the net area.*

**21.** $f(x) = 4 - 2x$ on $[0, 4]$

**22.** $f(x) = 8 - 2x^2$ on $[0, 4]$

**23.** $f(x) = \sin 2x$ on $\left[0, \frac{3\pi}{4}\right]$

**24.** $f(x) = x^3$ on $[-1, 2]$

**25.** $f(x) = \tan^{-1}(3x - 1)$ on $[0, 1]$

**26.** $f(x) = xe^{-x}$ on $[-1, 1]$

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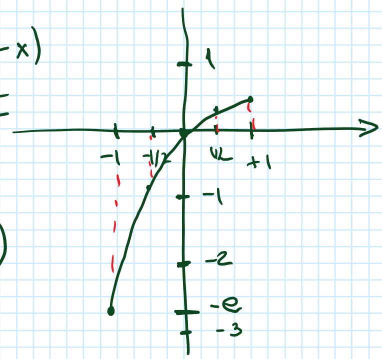
$f : [-1,1] \to \mathbb{R}$ ; $f(x) = xe^{-x}$ ; $f'(x) = e^{-x}(1-x)$

$n = 4 \Rightarrow \Delta x = \frac{1}{2}$

| $x$ | $-1$ | | $-\frac{1}{2}$ | | $0$ | | $\frac{1}{2}$ | | $1$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $-e$ | | $-\sqrt{e}/2$ | | $0$ | | $1/2\sqrt{e}$ | | $e^{-1}$ |
| $f'(x)$ | $2e$ | $+$ | $+$ | | $1$ | $+$ | $+$ | | $0$ |




$$\int_{-1}^{+1} f(x)\,dx \cong \Delta x\left(f(-1) + f(-\tfrac{1}{2}) + f(0) + f(\tfrac{1}{2})\right)$$

$$\cong \frac{1}{2}\left(-e - \frac{\sqrt{e}}{2} + 0 + \frac{1}{2\sqrt{e}}\right)$$

5.2.28.

**27–30. Area versus net area** *Graph the following functions. Then use geometry (not Riemann sums) to find the area and the net area of the region described.*

**27.** The region between the graph of $y = -3x$ and the $x$-axis, for $-2 \le x \le 2$

**28.** The region between the graph of $y = 4x - 8$ and the $x$-axis, for $-4 \le x \le 8$

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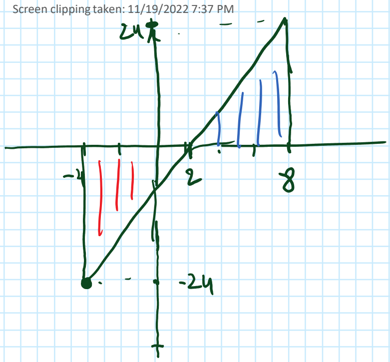


$y(x) = 4x - 8$ $\quad y(x) = 0 \Rightarrow x = 2$

$$\int_{-4}^{8} y(x)\,dx = \underbrace{\frac{1}{2}\Big(\overset{\text{base}}{6}\cdot\overset{\text{height}}{(-24)}\Big)}_{\text{Triangle area}} + \underbrace{\frac{1}{2}\Big(\overset{\text{base}}{6}\cdot\overset{\text{height}}{24}\Big)}_{\text{Triangle area}} = 0$$

$$\int_{-4}^{8} |y(x)|\,dx = 6 \cdot 24 = 144.$$

5.2.34.

**31–34. Approximating definite integrals** *Complete the following steps for the given integral and the given value of n.*

a. *Sketch the graph of the integrand on the interval of integration.*

b. *Calculate $\Delta x$ and the grid points $x_0, x_1, \ldots, x_n$, assuming a regular partition.*

c. *Calculate the left and right Riemann sums for the given value of n.*

d. *Determine which Riemann sum (left or right) underestimates the value of the definite integral and which overestimates the value of the definite integra*

**31.** $\int_3^6 (1 - 2x)\,dx$; $n = 6$

**32.** $\int_0^2 (x^2 - 2)\,dx$; $n = 4$

**33.** $\int_1^7 \frac{1}{x}\,dx$; $n = 6$

**34.** $\int_0^{\pi/2} \cos x\,dx$; $n = 4$

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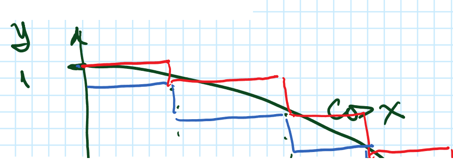
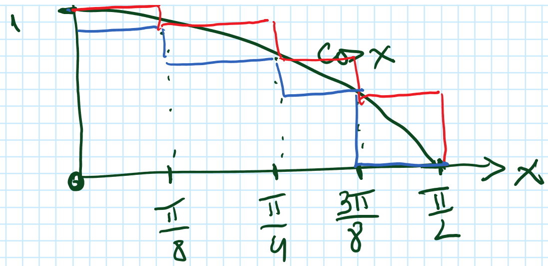
$$I = \int_0^{\pi/2} \cos x\,dx \;;\; n = 4$$



$I = \int_0^{\pi/2} \cos x \, dx \; ; \; n = 4$

$\Delta x = \frac{\pi}{8}$




$I_R$ = Right Riemann sum underestimates

$I_L$ = Left Riemann sum overestimates

$$I_R = \Delta x \left( \cos\frac{\pi}{8} + \cos\frac{\pi}{4} + \cos\frac{3\pi}{8} + \cos\frac{\pi}{2} \right)$$

$$I_L = \Delta x \left( \cos 0 + \cos\frac{\pi}{8} + \cos\frac{\pi}{4} + \cos\frac{3\pi}{8} \right).$$

5.2.43.

**39–46. Definite integrals** *Use geometry (not Riemann sums) to evaluate the following definite integrals. Sketch a graph of the integrand, show the region in question, and interpret your resul*

**39.** $\int_0^4 (8 - 2x)\, dx$

**40.** $\int_{-4}^2 (2x + 4)\, dx$

**41.** $\int_{-1}^2 (-|x|)\, dx$

**42.** $\int_0^2 (1 - x)\, dx$

**43.** $\int_0^4 \sqrt{16 - x^2}\, dx$

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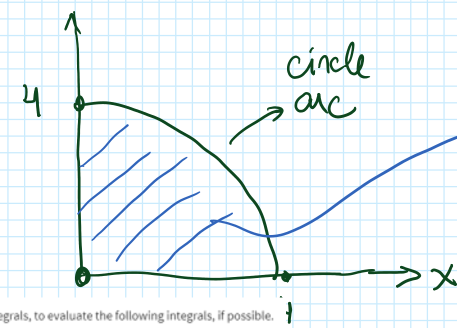
$I = \int_0^4 \sqrt{16 - x^2}\, dx$




$\frac{1}{4}$ area of circle

$r = 4$

$I = \frac{1}{4} \pi r^2 = 4\pi$

**51. Properties of integrals** Use only the fact that $\int_0^4 3x(4 - x)\, dx = 32$, and the definitions and properties of integrals, to evaluate the following integrals, if possible.

**a.** $\int_4^0 3x(4 - x)\, dx$

**b.** $\int_0^4 x(x - 4)\, dx$

**c.** $\int_4^0 6x(4 - x)\, dx$

**d.** $\int_0^8 3x(4 - x)\, dx$

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5.2.51. a) $\int_4^0 3x(4-x)\,dx = -\int_0^4 3x(4-x) = -32$

b) $\int_0^4 x(x-4)\,dx = -\frac{1}{3}\int_0^4 3x(4-x) = -\frac{32}{3}$

c) $\int_4^0 6x(4-x)\,dx = -2\int_0^4 3x(4-x) = -64$

d) $\int_0^8 3x(4-x)\,dx = \int_0^4 3x(4-x) + \int_4^8 3x(4-x)\,dx$

$= 32 + \int_4^8 3x(4-x)\,dx.$

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**69. Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample.

**a.** If $f$ is a constant function on the interval $[a, b]$, then the right and left Riemann sums give the exact value of $\int_a^b f(x)\, dx$, for any positive integer $n$.

**b.** If $f$ is a linear function on the interval $[a, b]$, then a midpoint Riemann sum gives the exact value of $\int_a^b f(x)\, dx$, for any positive integer $n$.

**c.** $\int_0^{2\pi/a} \sin ax\, dx = \int_0^{2\pi/a} \cos ax\, dx = 0$ (*Hint:* Graph the functions and use properties of trigonometric functions.)

**d.** If $\int_a^b f(x)\, dx = \int_b^a f(x)\, dx$, then $f$ is a constant function.

**e.** Property 4 of Table 5.4 implies that $\int_a^b x f(x)\, dx = x \int_a^b f(x)\, dx$.

5.2.69.

a) $f(x) = c \Rightarrow \int_a^b c\, dx = c(b - a)$

a rectangle.

Left Riemann = Right Riemann = Exact

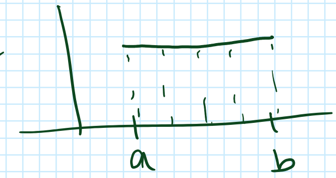
c

a rectangle.

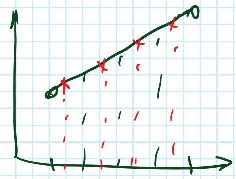
Left Riemann = Right Riemann = Exact

**e.** Property 4 of Table 5.4 implies that $\int_a^b x f(x)\,dx = x\int_a^b f(x)\,dx$.

c

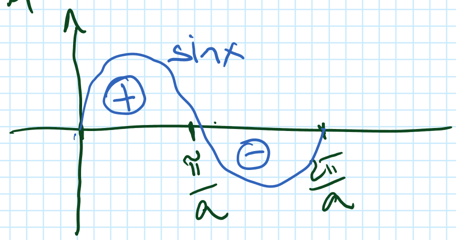


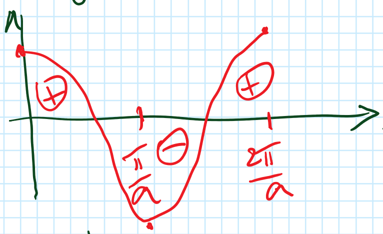

b)




Midpoint Riemann = exact = area of trapezoid

c) $\int_0^{2\pi/a} \sin ax\,dx = \int_0^{2\pi/a} \cos ax\,dx = 0$




Positive & negative areas cancel out.

d) $\int_a^b f(x)\,dx = \int_b^a f(x)\,dx = -\int_a^b f(x)\,dx \Rightarrow$

$2\int_a^b f(x)\,dx = 0 \Rightarrow \int_a^b f(x)\,dx = 0$

$f$ is not necessarily constant $\int_{-1}^{1} x\,dx = 0$ but $f(x) = x$ not constant

e) False $\int_a^b x f(x)\,dx$ is a constant

$x\int_a^b f(x)\,dx$ is a function of $x$

5.2.71

**70–74. Approximating definite integrals with a calculator** *Consider the following definite integrals.*

*a. Write the left and right Riemann sums in sigma notation for an arbitrary value of n.*

*b. Evaluate each sum using a calculator with n = 20, 50, and 100. Use these values to estimate the value of the integral.*

**70.** $\int_4^9 3\sqrt{x}\,dx$

**71.** $\int_0^1 (x^2+1)\,dx$

**72.** $\int_1^e \ln x\,dx$

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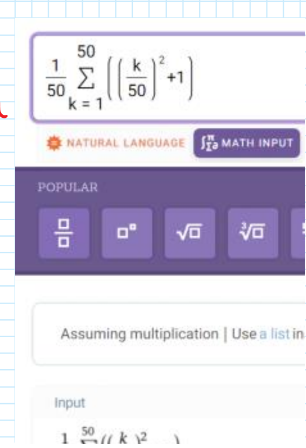
$I = \int_0^1 f(x)\,dx = \int_0^1 (x^2+1)\,dx \approx \Delta x \sum_{k=1}^{n} f(x_{k-1})$ Left sum

$I \approx \Delta x \sum_{k=1}^{n} f(x_k)$ Right sum

$x_k = k\Delta x, \quad k = 0, 1, \ldots, n$

$\Delta x = \frac{1}{n}$



5.2.94.

**94. Zero net area** Assuming $0 < c < d$, find the value of $b$ (in terms of $c$ and $d$) for which $\int_c^d (x+b)\,dx = 0$.

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$$I = \int_c^d (x+b)\,dx = \frac{1}{2}(d^2 - c^2) + b(d-c) \Rightarrow$$

$$I = (d-c)\left(\frac{d+c}{2} + b\right) = 0 \Rightarrow \quad b = -\frac{c+d}{2}$$

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Input

$$\frac{1}{50}\sum_{k=1}^{50}\left(\left(\frac{k}{50}\right)^2 + 1\right)$$

Result

$$\frac{6717}{5000} = 1.3434$$