Test 1 Solution

Tuesday, September 20, 2022 9:25 PM

Note: concise solution with motivation

1. Let $f(x) = \dfrac{|x-5|}{x^2-25}$ f undefined at $x=5$

For $x<5$ $f(x) = \dfrac{5-x}{x^2-25} = \dfrac{5-x}{(x-5)(x+5)} = -\dfrac{1}{x+5}$

For $x>5$ $f(x) = \dfrac{x-5}{x^2-25} = \dfrac{1}{x+5}$

$L_- = \lim\limits_{x\to 5_-} f(x) = -\dfrac{1}{10}$; $L_+ = \lim\limits_{x\to 5_+} f(x) = \dfrac{1}{10}$

Since one-sided limits are not equal $(L_- \neq L_+)$ the limit $\lim\limits_{x\to 5} f(x)$ does not exist.

2. Let $f(x) = x^3 - 5x^2 + 2x + 1$. Given equation is equivalent to $f(x)=0$

Evaluate at interval endpoints $f(-1) = -1-5-2+1 = -7 < 0$

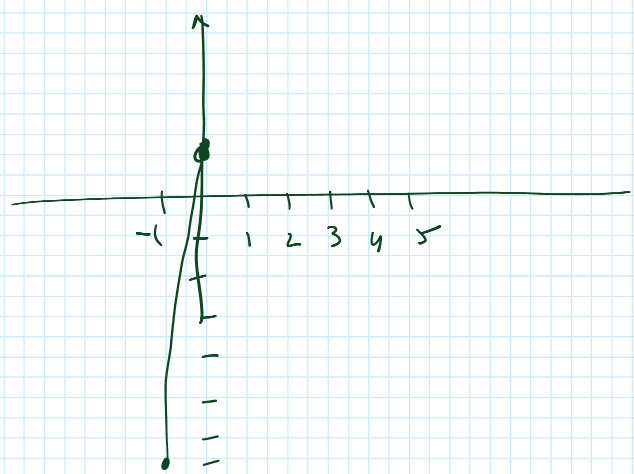
$f(5) = 125 - 125 + 10 + 1 = 11 > 0$

$f(x)$ is continuous and changes sign between $x=-1$ and $x=5$

Intermediate value theorem implies there exist some c such that $f(c)=0$ $-1 < c < 5$

$f(0) = 1$

Solution is between -1 & 0.



3. $f(x) = \dfrac{2}{3x+1}$ $a=-1$

$f(a) = \dfrac{2}{-3+1} = -1$

Tangent line has equation $y = mx + n$

$m = f'(a) = -\dfrac{2\cdot 3}{(3x+1)^2} = -\dfrac{6}{4} = -\dfrac{3}{2}$

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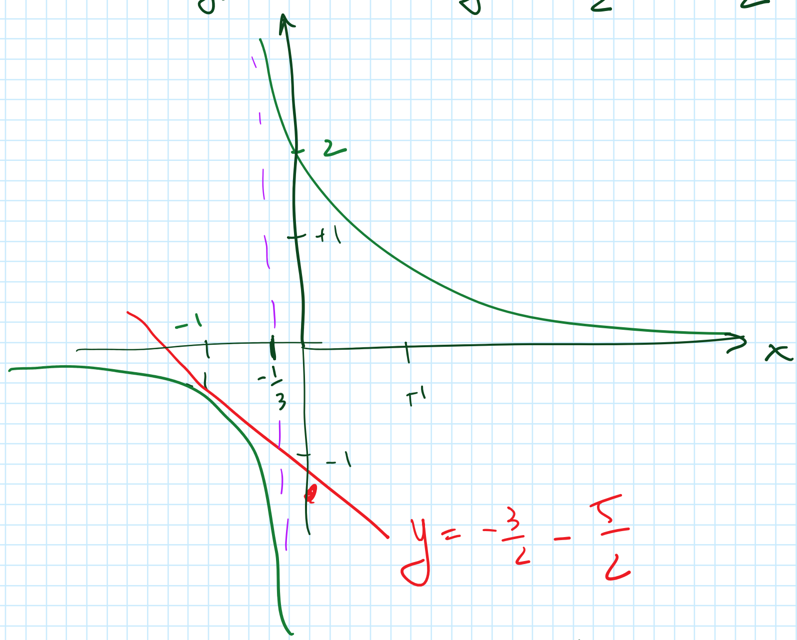
$$-1 = f(a) = m\cdot a + n = \left(-\frac{3}{2}\right)(-1) + n \Rightarrow n = -\frac{5}{2}$$

Tangent line $y = -\frac{3}{2}x - \frac{5}{2}$



4. $$L_+ = \lim_{x\to\infty} \frac{3e^{5x} + 7e^{6x}}{9e^{5x} + 14e^{6x}} = \lim_{x\to\infty} \frac{\cancel{e^{6x}}(3e^{-x} + 7)}{\cancel{e^{6x}}(9e^{-x} + 14)}$$

As $x \to \infty$ $e^{-x} \to 0$ hence

$$L_+ = \frac{7}{14} = \frac{1}{2}$$

$$L_- = \lim_{x\to -\infty} \frac{\cancel{e^{5x}}(3 + 7e^{x})}{\cancel{e^{5x}}(9 + 14e^{x})}$$

As $x \to -\infty$, $e^{x} \to 0$ hence $L_- = \frac{3}{9} = \frac{1}{3}$

5. $f(x) = g(x) + h(x)$ $\quad g(x) = (x^2+1)\cos x$

$h(x) = (x^2-1)\sin x$

$f' = g' + h'$; $\quad g(x) = a(x)\, b(x)$

Product rule $g' = a'b + ab'$

$$g'(x) = 2x\cos x - (x^2+1)\sin x$$

$$h'(x) = 2x\sin x + (x^2-1)\cos x$$

$$f'(x) = (x^2+2x-1)\cos x - (x^2-2x+1)\sin x$$

$$f''(x) = g''(x) + h''(x)$$; Again apply product rule

$$g''(x) = 2\cos x - 2x\sin x - 2x\sin x - (x^2+1)\cos x$$
$$= (1-x^2)\cos x - 4x\sin x$$

$$h''(x) = 2\sin x + 2x\cos x + 2x\cos x - (x^2-1)\sin x$$
$$= (3-x^2)\sin x + 4x\cos x$$

$$f''(x) = (1+4x-x^2)\cos x - (x^2+4x-3)\sin x$$