Tuesday, October 25, 2022 10:33 AM

1. Find the derivative $f'(z)$ of

$$f(z) = \left(\frac{z^2+1}{z}\right)e^z.$$

Are there points in the domain of f where f' does not exist?

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$f: \mathbb{R} \setminus \{0\} \to \mathbb{R}$

$f(z) = \left(z + \frac{1}{z}\right)e^z = g(z)\,h(z)$

Product rule. $f' = g'h + gh'$

$g'(z) = 1 - \frac{1}{z^2}$; $h'(z) = e^z$

$f'(z) = e^z\left[1 - \frac{1}{z^2} + z + \frac{1}{z}\right]$. ✓

$f': \mathbb{R} \setminus \{0\} \to \mathbb{R}$

2.

Determine the domain of the function $y(x) = e^{\sqrt{x}} + x^{\sqrt{e}}$, and find its derivative.

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Sum rule $\quad y'(x) = (e^{\sqrt{x}})' + (x^{\sqrt{e}})'$

$= e^{\sqrt{x}}\frac{1}{2\sqrt{x}} + \sqrt{e}\, x^{\sqrt{e}-1}$

Chain rule: $e^{\sqrt{x}} = h(g(x)) \quad h(u) = e^u \quad g(x) = \sqrt{x}$

$(e^{\sqrt{x}}) = h'(u)\,g'(x)$

3.

3. Find the derivative $x'(y)$ from the implicit function definition $\sin x \cos y = \sin x + \cos y$

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$\frac{d}{dy}$: $\quad \sin x \cos y = \sin x + \cos y \Rightarrow$

$(\cos x)\, x' \cos y + \sin x\,(-\sin y) = \cos x \cdot x' - \sin y$

$\cos x \cdot (\cos y - 1)\, x' = (\sin y)(\sin x - 1)$

$x' = \frac{\sin y\,(\sin x - 1)}{(\cos y - 1)\cos x}$

4. Molten metal in a cylinder of radius $r = 10$cm is pushed by a piston moving at velocity $v = 2$cm/s through a circular outlet of radius $a = 4$cm, and forms a growing hemisphere of radius R.

a) At what rate is the volume of the hemisphere increasing?

b) At what rate is the radius of the hemisphere increasing?

c) What is the velocity of the material through the outlet of radius a?

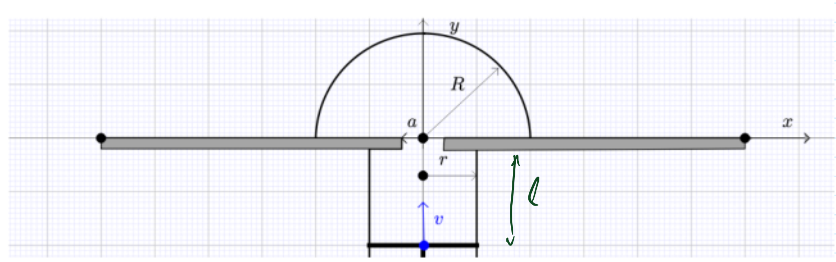

Figure 1.

Volume of hemi sphere:

$V(R) = \frac{2}{3}\pi R^3$; $R(t)$; $V(R(t))$

Volume of cylinder

$T(l) = \pi r^2 l \Rightarrow l(t)$

$T(l(t))$

$\frac{d}{dt} V = \frac{d}{dt} T \Rightarrow$

(1) $\frac{4}{3}\pi 3R^2 R' = \pi r^2 l' = \pi r^2 v$

Figure 1.

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(1) $\frac{4}{3}\pi 3R^2 R' = \pi r^2 \ell' = \pi r^2 v$

a) $V'(R(t)) = \frac{d}{dt} V(R(t)) = \pi r^2 v$

b) (1) $R' = \frac{1}{4} \cdot \frac{r^2 v}{R^2}$

c) Velocity in outlet w $\quad \frac{dT}{dt} = \pi r^2 v$ $\quad \Big| \quad \frac{dT}{dt} = \frac{dS}{dt} \Rightarrow$

Volume of outlet is S; $\quad \frac{dS}{dt} = \pi a^2 w$ $\quad \Big|$

$w = \left(\frac{r}{a}\right)^2 v$

5.) ...on $f(x) = 2x^2 \ln x - 5x^2$. Ide... on domain

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x	0		e		e^2		∞
f	0	↘	$-3e^2$		$-e^4$		$+\infty$
f'	0	$-$ $-$	$-$	$-$	0	$+$ $+$ $+$ $+$	∞
f''	$-\infty$	$-$ $-$ ∩	0	$+$	$+$ ∪	$+$	∞

$f' = \frac{2x^2}{x} + 4x \ln x - 10x = 4x \ln x - 8x = 4x(\ln x - 2)$

$f'' = 4(\ln x - 2) + 4x \frac{1}{x} = 4 \ln x - 4 = 4(\ln x - 1)$

Critical pts: $x_1 = 0$; $\ln x_2 = 2 \Rightarrow x_2 = e^2$

Inflection -||- : $\ln x_3 = 1 \Rightarrow x_3 = e$

$-3e^2 \cong (-3)(2.7)^2 \cong (-3)(3^2) = -27$

$-e^4 \cong 3^4 \cong -81$