

## MATH 231.FA22 TEST 2

Solve the following exercises explaining all solution steps. (45 minutes)

1. Find the derivative  $f'(z)$  of

$$f(z) = \left( \frac{z^2 + 1}{z} \right) e^z.$$

Are there points in the domain of  $f$  where  $f'$  does not exist?

2. Determine the domain of the function  $y(x) = e^{\sqrt{x}} + x^{\sqrt{e}}$ , and find its derivative.
3. Find the derivative  $x'(y)$  from the implicit function definition  $\sin x \cos y = \sin x + \cos y$
4. Molten metal in a cylinder of radius  $r = 10$  cm is pushed by a piston moving at velocity  $v = 2$  cm/s through a circular outlet of radius  $a = 4$  cm, and forms a growing hemisphere of radius  $R$ .
  - a) At what rate is the volume of the hemisphere increasing?
  - b) At what rate is the radius of the hemisphere increasing?
  - c) What is the velocity of the material through the outlet of radius  $a$ ?

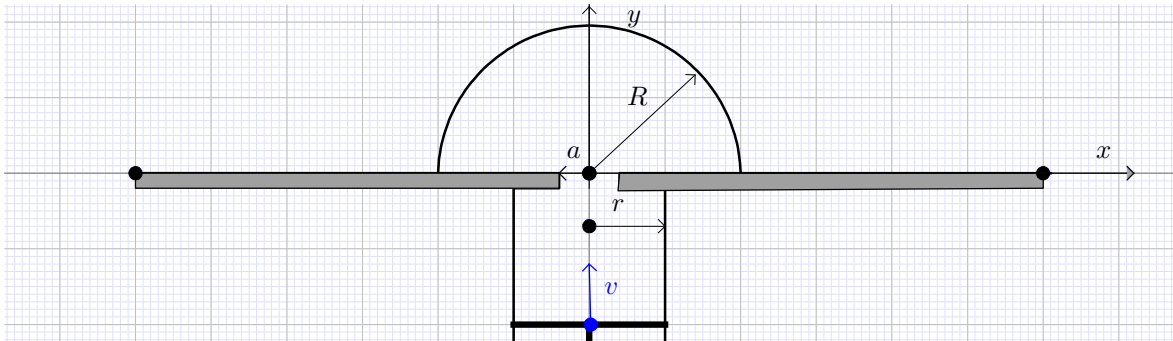


Figure 1.

5. Construct a qualitative graph of the function  $f(x) = 2x^2 \ln x - 5x^2$ . Identify roots, critical points, inflection points, behavior at limits of function domain.