MATH347.SP.01 Midterm Practice Examination

Instructions. *Answer the following questions. Provide a motivation of your approach and the reasoning underlying successive steps in your solution. Write neatly and avoid erasures. Use scratch paper to sketch out your answer for yourself, and then transcribe your solution to the examination you turn in for grading. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit. Each complete, correct solution to an examination question is awarded 3 course grade points. Your primary goal should be to demonstrate understanding of course topics and skill in precise mathematical formulation and solution procedures.*

1. Consider a vector $\boldsymbol{b} \in \mathbb{R}^{3\times 1}$ with components $b_1 = 1$, $b_2 = 2$, $b_3 = -1$ in the canonical basis

$$\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

Let x_1, x_2, x_3 denote the components of $\boldsymbol{b}$ with respect to the basis

$$\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3\} = \left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

Compute $\boldsymbol{x} = [\ x_1 \ \ x_2 \ \ x_3\ ]^T \in \mathbb{R}^{3\times 1}$. (3 points)

2. Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

a) Find bases for the subspaces $C(\boldsymbol{A})$, $C(\boldsymbol{A}^T)$, $N(\boldsymbol{A})$, $N(\boldsymbol{A}^T)$. (4 points)

b) Compute the orthogonal projection of the vector $\boldsymbol{b} = [\ 1 \ \ 1 \ \ 1\ ]^T$ onto $C(\boldsymbol{A})$. (2 points)

c) Compute the QR decomposition of $\boldsymbol{A}$. (3 points)