MATH347: Linear algebra for applications

# Homework 12

This assignment is a worksheet of exercises intended as preparation for the Final Examination. You should:

1. Review Lessons 13 to 24
2. Set aside 60 minutes to solve these exercises. Each exercise is meant to be solved within 3 minutes. If you cannot find a solution within 3 minutes, skip to the next one.
3. Check your answers in Matlab. Revisit theory for skipped or incorrectly answered exercies.

Per course policy, your best 10 homework scores enter into your final grade. Homeworks 11 and 12 are thus considered supplementary assignments. Nonetheless, everyone should attempt these as final examination preparation.

## 1 Matrix factorization

1. State $\boldsymbol{P} \in \mathbb{R}^{3\times 3}$ that permutes rows (1,2,3) of $\boldsymbol{A} \in \mathbb{R}^{3\times 3}$ as rows (2,3,1) through the product $\boldsymbol{PA}$.
2. Find the inverse of matrix $\boldsymbol{P}$ from Ex. 1.
3. State $\boldsymbol{Q} \in \mathbb{R}^{3\times 3}$ that permutes columns (1,2,3) of $\boldsymbol{A} \in \mathbb{R}^{3\times 3}$ as columns (3,1,2) through the product $\boldsymbol{AQ}$.
4. Find the inverse of marix $\boldsymbol{Q}$ from Ex. 3.
5. Find the $LU$ factorization of
$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}.$$
6. Find the $LU$ factorization of
$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$
7. Prove that permutation matrices $\boldsymbol{P}, \boldsymbol{Q}$ from Ex.1,3 are orthogonal matrices.
8. Find the $QR$ factorization of
$$\boldsymbol{A} = \begin{bmatrix} 0 & 5 & 6 \\ 0 & 0 & 9 \\ 1 & 2 & 3 \end{bmatrix}.$$
9. Find the eigendecomposition of $\boldsymbol{R} \in \mathbb{R}^{2\times 2}$, the matrix of reflection across the first bisector (the $x = y$ line).
10. Find the SVD of $\boldsymbol{R} \in \mathbb{R}^{2\times 2}$, the rotation by angle $\theta$ matrix.

## 2 Linear algebra problems

1. Find the coordinates of $\boldsymbol{b} = [\ 6 \ \ 15 \ \ 24\ ]^T$ on the $\mathbb{R}^3$ basis vectors
$$\left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}.$$
2. Solve the least squares problem $\min_{\boldsymbol{x}} \|\boldsymbol{b} - \boldsymbol{Ax}\|$ for
$$\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 3 & -5 \\ -11 & 21 \\ 0 & 0 \end{bmatrix}.$$
3. Find the line passing closest to points $\mathcal{D} = \{(-2,3), (-1,1), (0,1), (1,3), (3,7)\}$.

4. Find an orthonormal basis for $C(\boldsymbol{A})$ where

$$\boldsymbol{A} = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}.$$

5. With $\boldsymbol{A}$ from Ex. 4 solve the least squares problem $\min_{\boldsymbol{x}} \|\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}\|$ where

$$\boldsymbol{b} = \begin{bmatrix} -4 \\ -3 \\ 3 \\ 0 \end{bmatrix}.$$

6. What is the best approximant $\boldsymbol{c} \in C(\boldsymbol{A})$ ($\boldsymbol{A}$ from Ex. 4) of $\boldsymbol{b}$ from Ex. 5.

7. Find the eigenvalues and eigenvectors of

$$\boldsymbol{A} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

8. For $\boldsymbol{A}$ from Ex. 7 find the eigenvalues and eigenvectors of $\boldsymbol{A}^2$, $\boldsymbol{A}^{-1}$, $\boldsymbol{A} + 2\boldsymbol{I}$.

9. Is the following matrix diagonalizable?

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

10. Find the SVD of

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$