- New concepts:
  - Matrix equations
  - Simple linear systems

- A linear equation for unknowns $x_1, \ldots, x_n$ is of form

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1,$$

  i.e., a linear combination of $x_1, \ldots, x_n$ is set equal to some value (b_1)
- Multiple (m) linear equations form a linear system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

- The above is concisely stated as $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, $\boldsymbol{A} \in \mathbb{R}^{m \times n}$, $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{b} \in \mathbb{R}^m$
- Linear systems can have:
  - No solutions: $0 \cdot x = 1$; $x + y = 1$, $x + y = 2$
  - Unique solution: $x = 1$; $x + y = 1$, $x - y = 0$
  - Infinitely many solutions: $x + y = 1$, $2x + 2y = 2$.

- Some systems $\boldsymbol{A}\boldsymbol{x}=\boldsymbol{b}$, $\boldsymbol{A}\in\mathbb{R}^{m\times m}$ are easily solvable:
  - Diagonal systems $\boldsymbol{A}=\operatorname{diag}([\,a_{11}\ \ a_{22}\ \ \ldots\ \ a_{mm}\,])$. If $a_{ii}\neq 0$ for all i then

$$x_i=b_i/a_{ii}$$

  - Triangular systems, $\boldsymbol{A}$ is upper triangular if all elements beneath diagonal are zero, solvable by *back-substitution*

$$\boldsymbol{A}=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1m} \\ 0 & a_{22} & a_{23} & \ldots & a_{2m} \\ 0 & 0 & a_{33} & \ldots & a_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & a_{mm} \end{bmatrix}$$

Example:

$$\begin{cases} x+3y-2z=5 \\ 2y-6z=4 \\ 3z=6 \end{cases} \Rightarrow z=2,\, y=8,\, x=-15.$$