- New concepts:
  - Simplicia - simplest geometric object in m dimensions
  - Volumes of simplicia
  - Volumes of complicated objects

1. $d=1$, define an interval by $\{\boldsymbol{r}_1, \boldsymbol{r}_2\}$, with $\boldsymbol{r}_1, \boldsymbol{r}_2 \in \mathbb{R}$, $\boldsymbol{r}_1 = (x_1)$, $\boldsymbol{r}_2 = (x_2)$. The signed length of the interval is $l = x_2 - x_1$ and can be computed as

$$\Delta = \begin{vmatrix} 1 & 1 \\ x_1 & x_2 \end{vmatrix}$$

2. $d=2$, define a triangle by $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3\}$, with $\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \in \mathbb{R}^2$. The signed area is

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

3. $d=3$, define a tetrahedron by $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4\}$. The signed volume is

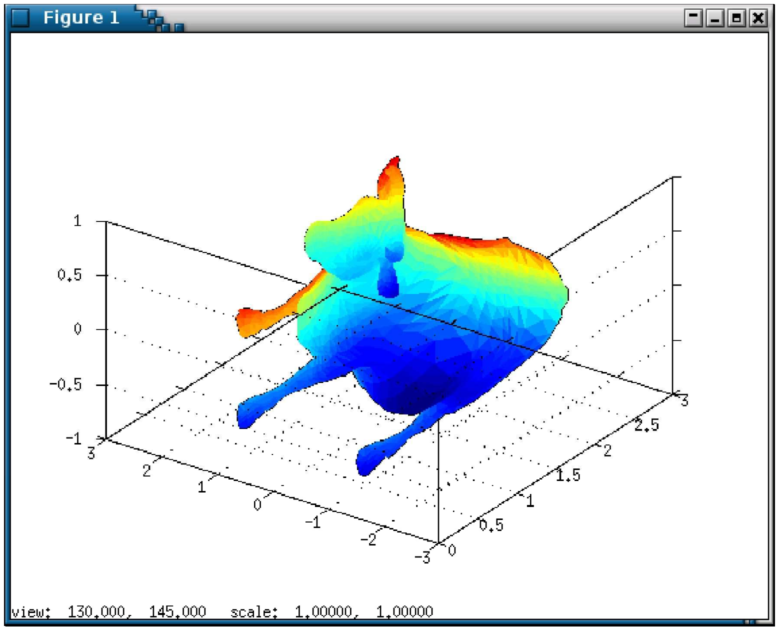
$$\Delta = \frac{1}{6} \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{vmatrix}$$

```
octave> cd ~/courses/MATH347/ply;
```

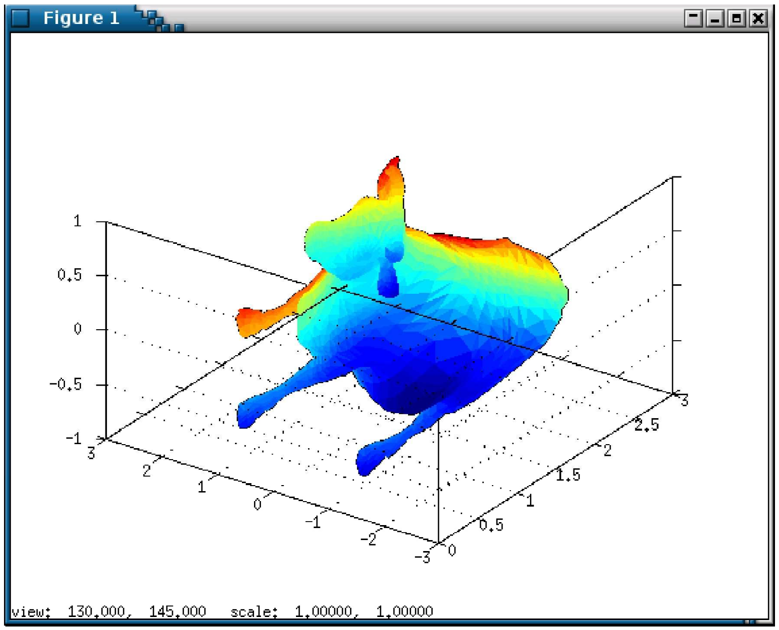
```
octave> [x,tri]=ply_to_tri_mesh('cow.ply'); tri=transpose(tri);
```

```
octave> trisurf(tri, x(1,:), x(2,:), x(3,:) ); print -dpng cow.png;
```

```
octave> tri(:,100)
```



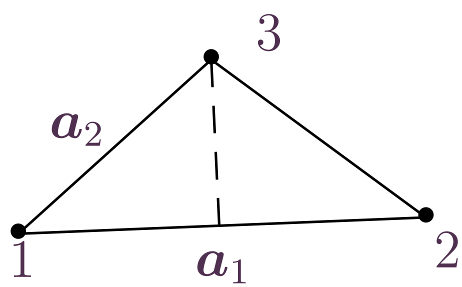
```
octave> cd ~/courses/MATH347/ply;
octave> [x,tri]=ply_to_tri_mesh('cow.ply'); tri=transpose(tri);
octave> trisurf(tri, x(1,:), x(2,:), x(3,:) ); print -dpng cow.png;
octave> tri(:,100)
ans =

    88    87   101
```



```
octave> Area=0.;
```

```
octave> for n=1:max(size(tri))
          n1=tri(1,n); n2=tri(2,n); n3=tri(3,n);
          a1 = x(:,n2)-x(:,n1); a2 = x(:,n3)-x(:,n1);
          q1 = a1/norm(a1); h = a2 - (q1'*a2)*q1;
          base = norm(a1); height = norm(h);
          Area = Area + 0.5*base*height;
        end;
```

```
octave> Area
```



$$\text{Area} = \frac{1}{2}(\text{base}) \times (\text{height})$$

$$A = \frac{1}{2}\|\boldsymbol{a}_1\|\,\|\boldsymbol{h}\| \quad \boldsymbol{q}_1 = \frac{1}{\|\boldsymbol{a}_1\|}\boldsymbol{a}_1$$

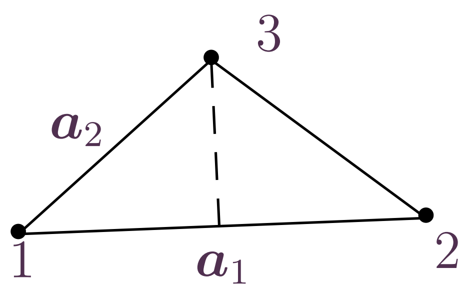
$$\boldsymbol{h} = \boldsymbol{a}_2 - (\boldsymbol{q}_1^T\boldsymbol{a}_2)\boldsymbol{q}_1$$

```
octave> Area=0.;
```

```
octave> for n=1:max(size(tri))
          n1=tri(1,n); n2=tri(2,n); n3=tri(3,n);
          a1 = x(:,n2)-x(:,n1); a2 = x(:,n3)-x(:,n1);
          q1 = a1/norm(a1); h = a2 - (q1'*a2)*q1;
          base = norm(a1); height = norm(h);
          Area = Area + 0.5*base*height;
        end;
```

```
octave> Area
```

```
Ahide =  21.193
```



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$$A = \frac{1}{2}\|\boldsymbol{a}_1\|\,\|\boldsymbol{h}\| \quad \boldsymbol{q}_1 = \frac{1}{\|\boldsymbol{a}_1\|}\boldsymbol{a}_1$$

$$\boldsymbol{h} = \boldsymbol{a}_2 - (\boldsymbol{q}_1^T\boldsymbol{a}_2)\boldsymbol{q}_1$$

```
octave> tet=delaunay(x(1,:),x(2,:),x(3,:));
```

```
octave> Volume=0.;
```

```
octave> for n=1:max(size(tet))
          n1=tet(n,1); n2=tet(n,2); n3=tet(n,3); n4=tet(n,4);
          X = [x(:,n1) x(:,n2) x(:,n3) x(:,n4)];
          A = [ones(1,4); X];
          Volume = Volume + abs(det(A))/6.;
        end;
```

```
octave> Volume
```

$$\text{Volume} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{vmatrix}$$

```
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          X = [x(:,n1) x(:,n2) x(:,n3) x(:,n4)];
          A = [ones(1,4); X];
          Volume = Volume + abs(det(A))/6.;
        end;
```

```
octave> Volume
```

```
Volume =  10.647
```

$$\text{Volume} = \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{vmatrix}$$

- Medical imaging (CT-scans) leads to extensive geometrical information

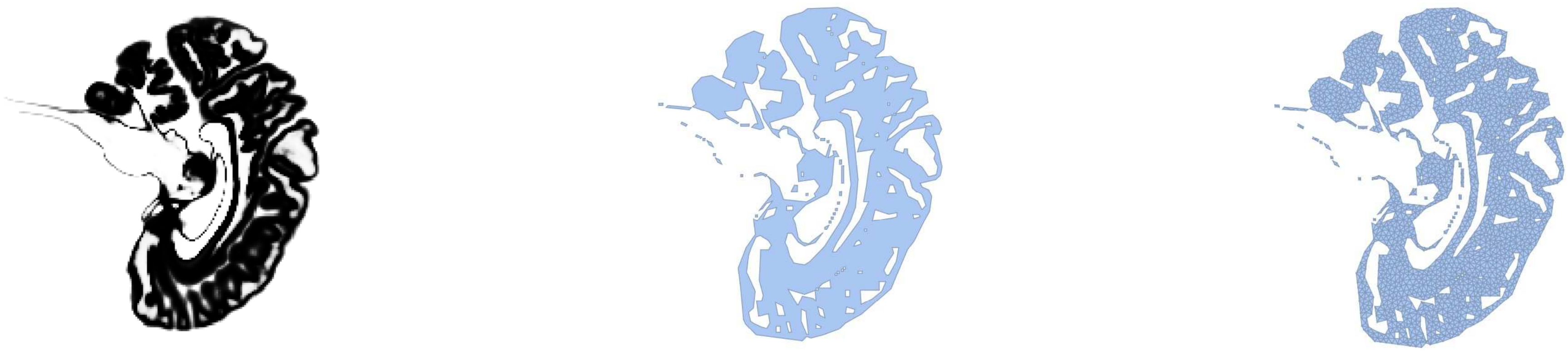

Figure 1. Left: CT scan of brain slice. Center: Brain regions. Right: Regions triangulation

- Periodic high-resolution CT-scans are processed to evaluate brain volume as needed for instance in Alzheimer monitoring