MATH347: Linear Algebra for Applications

Multidimensional quantities abound in nature and are ubiquituous in data science. The simplest procedure to construct multidimensional quantities is through linear combinations. Linear algebra furnishes the framework to define and characterize linear combinations and linear mappings. Because a complete characterization is furnished for finite dimensional systems, linear algebra finds wide applicability throughout all fields of study, including local approximation of nonlinear behavior.

- Linear combinations, matrix-vector product, matrix-matrix product
- Measuring vectors: scalar product and norm
- Vector spaces $\mathbb{Z}^n, \mathbb{R}^n, \mathbb{C}^n$
- Matrix range, linear dependence, independence, orthogonality
- Vector subspaces, left null space
- Vector space basis, sum and intersection of vector spaces
- Matrix subspaces, relations between subspaces, rank-nullity
- The basic problems within linear algebra: change of basis (linear systems), subspace projection (least squares), colinear linear combinations (eigenrelation)
- LU factorization to compute coordinates in a new basis
- QR factorization to compute an orthogonal column space basis
- Projection and least squares
- QR factorization of monomials, the orthogonal polynomials
- Eigenrelation and eigendecomposition
- Singular value decomposition, low-rank approximation