MATH347.SP.01 Midterm Examination

Instructions. *Answer the following questions. Provide concise motivation of your approach. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit.*

1. Compute the projection of $\boldsymbol{b}$ onto the column space of $\boldsymbol{A}$, (2 points)

$$\boldsymbol{b} = \begin{bmatrix} 49 \\ 49 \\ 49 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 3 & -6 \\ 6 & 2 \end{bmatrix}.$$

2. Let $R: \mathbb{R}^2 \to \mathbb{R}^2$ denote rotation by angle θ. Let $S: \mathbb{R}^2 \to \mathbb{R}^2$ denote a stretching transformation by α along the x_1 axis, and β along the x_2 axis. Let $T = R \circ S$ denote the composite transformation of stretching followed by rotation. (3 points)

   a) Write the matrix $\boldsymbol{A}$ representing R.

   b) Write the matrix $\boldsymbol{B}$ representing S.

   c) Write the matrix $\boldsymbol{C}$ representing T.

3. Consider (5 points)

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   a) What is the rank of $\boldsymbol{A}$?

   b) State a basis for $C(\boldsymbol{A})$.

   c) State a basis for $N(\boldsymbol{A}^T)$.

   d) State a basis for $C(\boldsymbol{A}^T)$.

   e) State a basis for $N(\boldsymbol{A})$.