MATH347.SP.25 Midterm Examination

Instructions. *Answer the following questions. Provide concise motivation of your approach. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit. Each correct question answer is awarded 2 course points.*

1. Prove the trigonometric identity

$$\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta).$$

Notation: $\cos^2(\theta) = [\cos(\theta)]^2$, $\sin^2(\theta) = [\sin(\theta)]^2$.

2. Determine the standard matrix $\boldsymbol{P}$ of the orthogonal projection of a vector $\boldsymbol{v} \in \mathbb{R}^3$ onto the line $x_1 = x_2 = x_3$.

3. Determine bases for the fundamental subspaces of the matrix $\boldsymbol{P}$ defined above.

4. Find the inverse of the standard matrix $\boldsymbol{M} \in \mathbb{R}^{3\times 3}$ of the linear mapping, $L = F \circ G$ with

a) $F: \mathbb{R}^3 \to \mathbb{R}^3$ denoting reflection across the vector $\boldsymbol{u} = [\ 1 \ \ 1 \ \ 0\ ]^T$;

b) $G: \mathbb{R}^3 \to \mathbb{R}^3$ denoting scaling by $\lambda_1, \lambda_2, \lambda_3$ along directions x_1, x_2, x_3, respectively.

5. Compute the LU factorization without permutations of

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 2 \\ 0 & -8 & -4 \end{bmatrix}.$$

Explicitly state the elementary matrices used at each stage of the process.