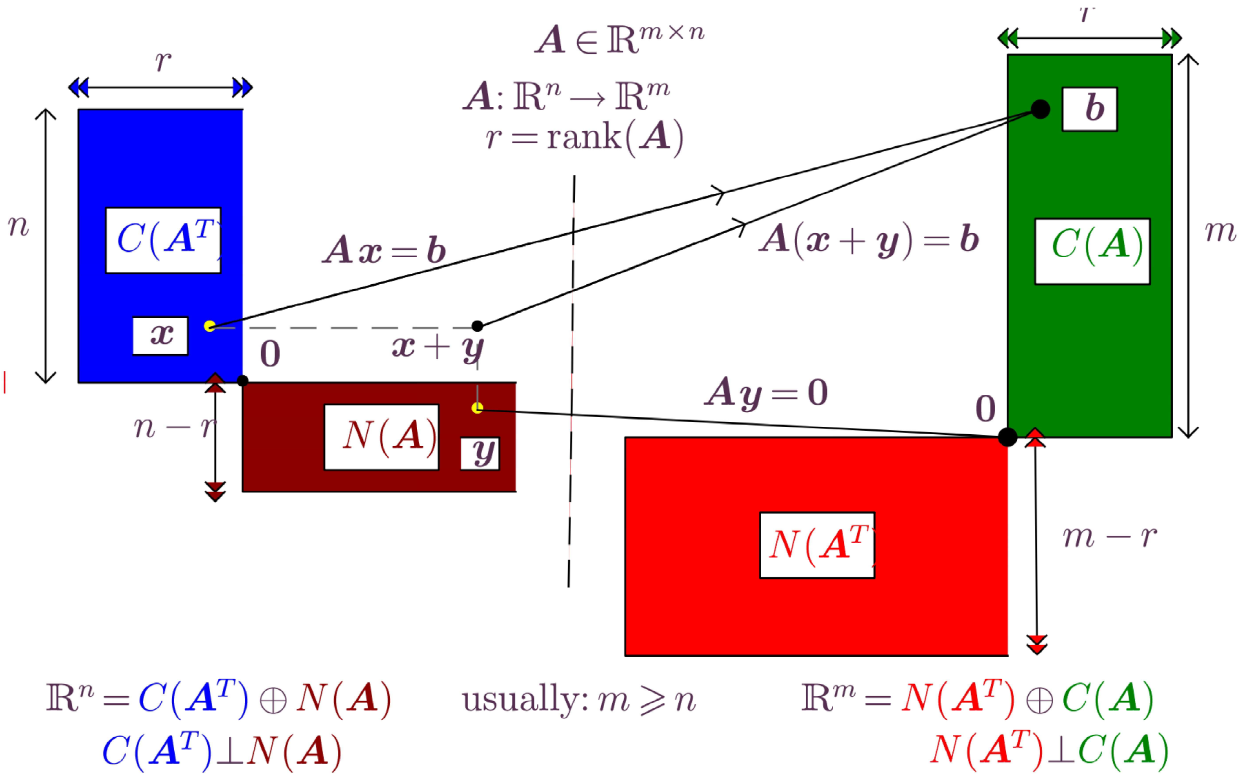
1 Matlab/Octave **orth** and **null** functions

The FTLA states a decomposition of the domain and codomain of a linear mapping T: $\mathbb{R}^n \rightarrow \mathbb{R}^m$, $T(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x}$.



In applications bases for the four fundamental spaces are also required. Recall that a basis a linearly independent spanning set for a vector space. In Matlab/Octave, a basis for $C(\boldsymbol{A})$ is given by `orth(A)`, and a basis for $N(\boldsymbol{A})$ is givrn by `null(A)`.

Here are some examples.

1.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & -1 \end{bmatrix}$$

The output from orth(A):

2.1615e-15	1.0000e+00	-2.0106e-16
8.1124e-01	-1.6364e-15	5.8471e-01
-5.8471e-01	1.1913e-15	8.1124e-01

Replacing small quantities with zero (inexact computer arithmetic) gives:

0	1.0000e+00	0

8.1124e-01	0	5.8471e-01
-5.8471e-01	0	8.1124e-01

There are 3 linearly independent vectors in orth(A), hence rank(A)=3, and the system $Ax=b$ has a unique solution for any b. The null space $N(A)$ should only contain the zero vector. Indeed, null(A) in Octave returns:

ans = []

Read the above as stating "there are no linearly independent vectors that span N(A)".

2.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

The output from orth(A) is

0.935113	-0.186157
0.342275	0.254295
-0.091712	-0.949041

There are now two linearly independent (column) vectors, hence rank(A)=2. By the FTLA the dimension of $N(A)$ should be 1, and indeed null(A) now gives:

-0.5774

-0.5774

0.5774

The system $Ax=b$ has an infinite number of solutions for $b \in C(A)$. Since vectors in orth(A) span $C(A)$, any linear combination of these vectors gives such a b, for example

$$b = \begin{bmatrix} 0.935113 \\ 0.342275 \\ -0.091712 \end{bmatrix}$$

If b is within $N(A^T)$, the system does not have a solution. Compute null(transpose(A)) to obtain

-0.3015

0.9045

0.3015

Any scaling of the above vector gives a rhs for which the system has no solution.

3.

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 & 2 \\ 2 & 1 & 3 & 4 & 3 \end{bmatrix}$$

orth(A) gives

0.2534 0.9674

0.9674 -0.2534

with two linearly independent vectors, hence rank(A)=2. Computing null(A) gives

2.7997e-01 -4.3303e-01 -7.5673e-01

-8.5573e-01 -4.8086e-01 -7.0125e-03

3.0834e-01 -2.2673e-01 2.0253e-01

-2.6872e-01 6.8707e-01 -2.1040e-01

1.4855e-01 -2.4039e-01 5.8483e-01

with 3 linearly independent vectors. The system Ax=b always has an infinite number of solutions.

4.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & 3 \\ 2 & 0 \end{bmatrix}$$

orth(A) gives

-0.495688 -0.510213

0.264100 0.050401

-0.824813 0.258209

0.065024 -0.818823

with 2 linearly independent vectors, hence rank(A)=2.

null(transpose(A)) gives

-0.019026 -0.702577

0.930667 -0.248134

0.322906   0.385673

0.170966   0.544125

and null(A) contains only the zero vector.

For $b = \begin{bmatrix} -0.495688 \\ 0.264100 \\ -0.824813 \\ 0.065024 \end{bmatrix}$ the system has a unique solution, while for $b = \begin{bmatrix} -0.019026 \\ 0.930667 \\ 0.322906 \\ 0.170966 \end{bmatrix}$ the system does not have any solution.