Practice Final Examination

Solve the following problems (5 course points each). Present a brief motivation of your method of solution. Problems 9 and 10 are optional; attempt them if you wish to improve your midterm examination score.

1. State the matrix product to obtain 3 linear combinations of vectors

$$\boldsymbol{u}=\begin{bmatrix}1\\0\\1\end{bmatrix}, \boldsymbol{v}=\begin{bmatrix}-1\\0\\1\end{bmatrix},$$

with scaling coefficients $(\alpha_1,\beta_1)=(1,1)$, $(\alpha_2,\beta_2)=(-1,1)$, $(\alpha_3,\beta_3)=(1,-1)$.

2. Orthonormalize the vectors

$$\boldsymbol{u}=\begin{bmatrix}1\\0\\1\end{bmatrix}, \boldsymbol{v}=\begin{bmatrix}-1\\0\\1\end{bmatrix}, \boldsymbol{w}=\begin{bmatrix}1\\1\\-1\end{bmatrix}.$$

3. For $x,y\in\mathbb{R}$, expansion of $(x-y)^3$ leads to $(x-y)^3=x^3-3x^2y+3xy^2-y^3$. Find the corresponding expansion of $(\boldsymbol{A}-\boldsymbol{B})^3$ for $\boldsymbol{A},\boldsymbol{B}\in\mathbb{R}^{m\times m}$.

4. Find the projection of $\boldsymbol{b}$ onto $C(\boldsymbol{A})$ for

$$\boldsymbol{b}=\begin{bmatrix}1\\2\\3\end{bmatrix}, \boldsymbol{A}=\begin{bmatrix}1 & -1\\1 & 0\\1 & 1\end{bmatrix}.$$

5. Find the LU decomposition of

$$\boldsymbol{A}=\begin{bmatrix}1 & 1 & 1\\2 & 3 & 3\\3 & 5 & 6\end{bmatrix}.$$

6. State the eigenvalues and eigenvectors of $\boldsymbol{R}\in\mathbb{R}^{2\times 2}$, the matrix describing reflection across the vector $\boldsymbol{w}=[\ 1\ \ 2\ ]^T$.

7. Compute the eigendecomposition of

$$\boldsymbol{A}=\begin{bmatrix}5/2 & 0 & 1/2\\0 & 1 & 0\\1/2 & 0 & 5/2\end{bmatrix}.$$

8. Find the SVD of

$$\boldsymbol{A}=\begin{bmatrix}1 & -1\\0 & 0\\1 & 1\end{bmatrix}.$$

9. Find the matrix of the reflection of $\mathbb{R}^2$ vectors across the vector $\boldsymbol{u}=[\ 1\ \ 2\ ]^T$.

10. Find bases for the four fundamental spaces of

$$\boldsymbol{A}=\begin{bmatrix}1 & -1\\0 & 0\\1 & 1\end{bmatrix}.$$