# Overview

- Theorem on existence of solution to IVP
- Examples

**Theorem.** *If* $f$ *continuous on the open rectangle* $R\colon a < x < b, c < y < d$, *and* $(x_0, y_0) \in R$, *then the IVP* $y' = f(x, y)$, $y(x_0) = x_0$ *has at least one solution on some open interval of* $(a, b)$ *that contains* $x_0$

**Theorem.** *If* $f$ *and* $f_y = \partial f / \partial y$ *continuous on the open rectangle* $R\colon a < x < b$, $c < y < d$, *and* $(x_0, y_0) \in R$, *then the IVP* $y' = f(x, y)$, $y(x_0) = x_0$ *has an unique solution on some open interval of* $(a, b)$ *that contains* $x_0$

# Example 1

$$y' = \frac{x^2 - y^2}{1 + x^2 + y^2}, \; y(x_0) = y_0$$

# Example 2

$$y' = \frac{x^2 - y^2}{x^2 + y^2}, \; y(x_0) = y_0$$

# Example 3

$$y' = \frac{x+y}{x-y}, \; y(x_0) = y_0$$

# Example 4

$$y' = 2x\,y^2,\; y(x_0) = y_0$$

# Example 5

$$y' = \frac{10}{3}\, x\, y^{2/5},\; y(x_0) = y_0$$

# Example 6

$$y' = \frac{10}{3}\, x\, y^{2/5},\; y(0) = 0$$

# Example 7

$$y' = \frac{10}{3}\,x\,y^{2/5},\, y(0) = -1$$

# Example 8

$$y' = \frac{10}{3}\,x\,y^{2/5},\ y(0) = 1$$