Overview

- Growth/decay models
- Heat transfer
- Mechanics

- How might the quantity $Q(t)$ change in time?
  - Rate of change determined by the independent variable t: $Q' = f(t)$

$$Q(t) = Q_0 + \int_0^t f(\tau)\,\mathrm{d}\tau = f(0)\,t + \frac{1}{2} f'(0) t^2 + \dots + \frac{1}{n!} f^{(n-1)}(0) t^n + \dots$$

  - Rate of change determined by the dependent variable Q: $Q' = g(Q)$

$$g(Q) \cong g(0) + g'(0)\,Q \Rightarrow Q(t) = e^{g'(0)\,t} Q_0 + (e^{g'(0)t} - 1)\frac{g(0)}{g(1)}$$

  - Rate of change determined by both variables: $Q' = h(t, Q)$

$$h(t, Q) \cong h(0,0) + h_t(0,0)t + h_Q(0,0)\,Q \equiv h_{00} + h_{10}\,t + h_{01}\,Q \Rightarrow$$

$$Q(t) = e^{h_{01}t} Q_0 + (e^{h_{01}t} - 1)\frac{h_{00}}{h_{01}} + (e^{h_{01}t} - 1 - h_{01}t)\frac{h_{10}}{h_{01}^2}$$

Growth rate depends only on time $\Rightarrow$ future system states accumulate past growth

$$Q(t) = Q_0 + \int_0^t f(t)\,\mathrm{d}t,\; f(t) = f_0 + f_1 t + f_2 t^2 + \cdots$$

Example 1. Constant growth rate $\Rightarrow$ linear growth, $Q(t) = Q_0 + f_0\, t$

Example 2. Linearly-varying growth rate $\Rightarrow$ quadratic growth

$$f(t) = f_0 + f_1\, t \Rightarrow Q(t) = Q_0 + f_0\, t + \frac{1}{2}\, f_1\, t$$

Example 3. Oscillatory growth rate $\Rightarrow$ delayed, oscillatory system state

$$f(t) = a \sin t \Rightarrow Q(t) = Q_0 + a\,(1 - \cos t)$$

Plots of $Q' = f(t)$ examples

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GNUplot] set xrange [0:6.28]; plot 1+.15*x,1+.15*x+.01*x**2,1+1-cos(x)
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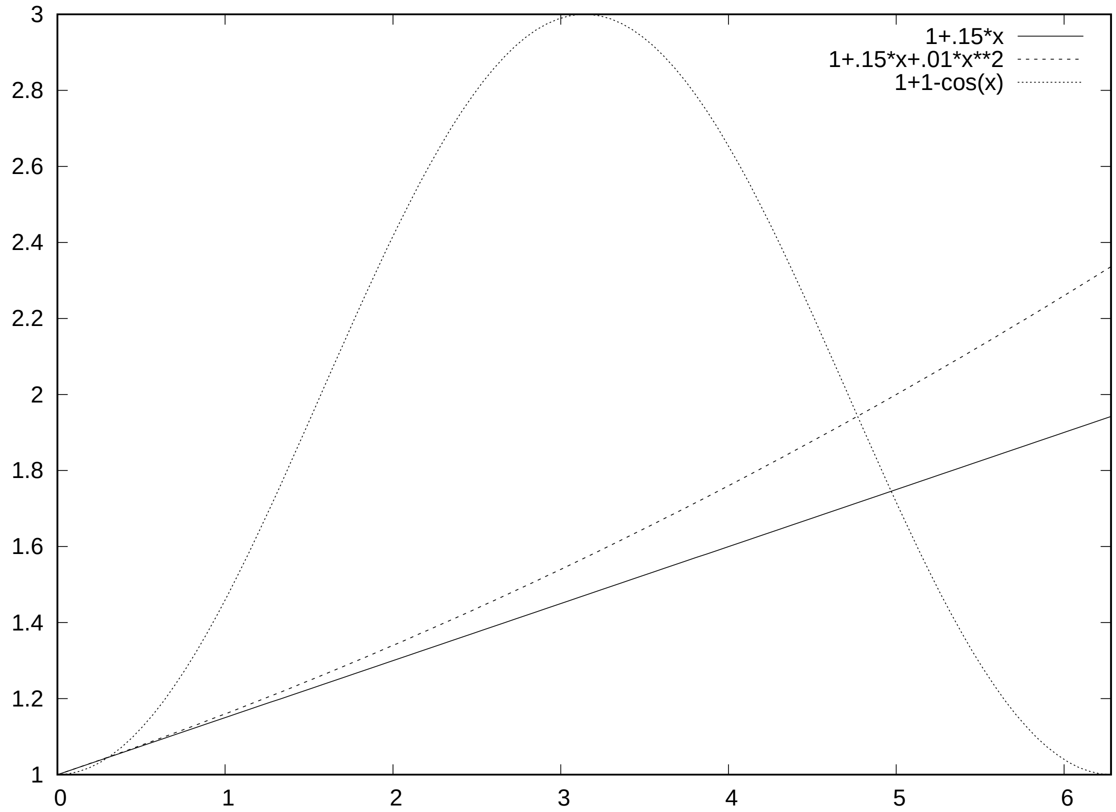


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GNUplot]
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Exponential growth decay

- If $Q' = g(Q)$, i.e., growth rate determined by dependent variable, the system exhibits exponential growth or decay $Q' = rQ, Q(t_0) = Q_0 \Rightarrow Q(t) = e^{r(t-t_0)}Q_0$
  - Radioactive decay $(r < 0)$
  - Compound interest accumulation
  - Carbon dating $r = -(\ln 2)/5570$ (half life of ^{14}C is 5570 years)
- Equilibrium temperature $T' = -k(T - T_m)$ (T_m ambient temperature)

$$T(t) = T_m + (T_0 - T_m)e^{-kt}.$$

- Mixing problems $Q' = r - cQ$

$$Q(t) = \frac{r}{c} + e^{-ct}\left(Q_0 - \frac{r}{c}\right).$$

- Conservation of momentum

$$\frac{\mathrm{d}}{\mathrm{d}t}(mv) = F \Rightarrow v' = \frac{F}{m}.$$

- Examples:

→ Object falling with air resistance $v' = g - cv$ (very low speed behavior)

$$v(t) = \frac{g}{c} + e^{-ct}\left(v_0 - \frac{g}{c}\right).$$

→ Object falling with air resistance $v' = g - dv^2$