Homework 6 Solution

Due date: Feb 20, 2020, 11:55PM.

Bibliography: Lesson09.pdf Lesson10.pdf. The first exercise in each problem set is solved for you to use as a model.

1. Consider $\mathcal{V} = \mathbb{R}^n$, $\mathcal{S} = \mathbb{R}$. Determine whether the column vectors of $\boldsymbol{A}$ form a basis for $\mathbb{R}^n$.

Ex 1. $n = 3$,

$$\boldsymbol{A} = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Solution. Reduce $\boldsymbol{A}$ to row-echelon form

$$\boldsymbol{A} \sim \begin{pmatrix} -1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & \dfrac{3}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

Since the row-echelon form does not contain a row of zeros, the columns of $\boldsymbol{A}$ form a basis for $\mathbb{R}^3$. Check in Maxima

```
(%i1) A: matrix([-1, 1, 1],[2, 0, 1],[1, 1, 1]);
```

(%o1) $\begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

```
(%i2) echelon(A);
```

(%o2) $\begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \end{pmatrix}$

```
(%i3)
```

Note: multiple row-echelon forms are possible (differing by, say, permutation of rows), but the same conclusion on the independence is reached, i.e., no zero rows implies linear independence.

Ex 2. $n = 3$,

$$\boldsymbol{A} = \begin{pmatrix} 2 & 5 & 3 \\ -2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}.$$

Solution. Reduce $\boldsymbol{A}$ to row-echelon form

$$\boldsymbol{A} \sim \begin{pmatrix} 2 & 5 & 3 \\ 0 & 6 & 4 \\ 0 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 2 & 5 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & \dfrac{5}{2} & \dfrac{3}{2} \\ 0 & 1 & \dfrac{2}{3} \\ 0 & 0 & 1 \end{pmatrix}$$

Since the row-echelon form does not contain a row of zeros, the columns of $\boldsymbol{A}$ form a basis for $\mathbb{R}^3$. Check in Maxima

```
(%i39) A: matrix([2, 5, 3],[-2, 1, 1],[1, 2, 1]);
```

(%o39) $\begin{pmatrix} 2 & 5 & 3 \\ -2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

(%i40) `echelon(A);`

(%o40) $\begin{pmatrix} 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{pmatrix}$

(%i41)

Ex 3. $n=4$,

$$\boldsymbol{A}=\begin{pmatrix} 1 & 2 & 2 & -1 \\ 1 & 1 & 4 & 2 \\ -1 & 3 & 2 & 0 \\ 1 & 1 & 5 & 3 \end{pmatrix}.$$

Solution. Reduce $\boldsymbol{A}$ to row-echelon form

$$\boldsymbol{A}\sim\begin{pmatrix} 1 & 2 & 2 & -1 \\ 0 & -1 & 2 & 3 \\ 0 & 5 & 4 & -1 \\ 0 & -1 & 3 & 4 \end{pmatrix}\sim\begin{pmatrix} 1 & 2 & 2 & -1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 14 & 14 \\ 0 & 0 & 1 & 1 \end{pmatrix}\sim\begin{pmatrix} 1 & 2 & 2 & -1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The row-echelon form does contain a row of zeros, the columns of $\boldsymbol{A}$ do not form a basis for $\mathbb{R}^4$. Check in Maxima

(%i41) `A: matrix([1, 2, 2, -1],[1, 1, 4, 2],[-1, 3, 2, 0],[1, 1, 5, 3]);`

(%o41) $\begin{pmatrix} 1 & 2 & 2 & -1 \\ 1 & 1 & 4 & 2 \\ -1 & 3 & 2 & 0 \\ 1 & 1 & 5 & 3 \end{pmatrix}$

(%i42) `echelon(A);`

(%o42) $\begin{pmatrix} 1 & -3 & -2 & 0 \\ 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

(%i43)

Ex 4. $n=4$,

$$\boldsymbol{A}=\begin{pmatrix} -1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 2 & -1 & 2 \end{pmatrix}.$$

Solution. Having understood the procedure of using row operations to obtain the reduced row-echelon form, resort not to automated computation and obtain in Maxima

(%i43) `A: matrix([-1, 2, 1, 2],[1, 1, 3, 1],[0, -1, 1, 1],[1, 2, -1, 2]);`

(%o43) $\begin{pmatrix} -1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 2 & -1 & 2 \end{pmatrix}$

```
(%i44) echelon(A);
```

$$\text{(\%o44)} \quad \begin{pmatrix} 1 & -2 & -1 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i45)
```

The row-echelon form does not contain any row of zeros, the columns of $\boldsymbol{A}$ form a basis for $\mathbb{R}^4$.

2. Determine whether the following column vectors of $\boldsymbol{B}$ form a basis for $(\mathcal{V}, +, \mathbb{R}, \cdot)$

Ex 1. $\boldsymbol{B} = \begin{pmatrix} 1 & 2x^2+x+2 & -x^2+x \end{pmatrix}$, $\mathcal{V} = \mathcal{P}_2$

Solution. Consider an arbitrary $\boldsymbol{p} \in \mathcal{V} = \mathcal{P}_2$

$$\boldsymbol{p}(x) = a_0 + a_1 x + a_2 x^2,$$

and check if $\boldsymbol{p}$ can be expressed as a linear combination of the basis vectors, $\boldsymbol{B}$, i.e., there exists $\boldsymbol{c} \in \mathbb{R}^3$ such that $\boldsymbol{B}\boldsymbol{c} = \boldsymbol{p}$

$$\boldsymbol{B}\boldsymbol{c} = \begin{pmatrix} \boldsymbol{b}_1(x) & \boldsymbol{b}_2(x) & \boldsymbol{b}_3(x) \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = c_1 \boldsymbol{b}_1(x) + c_2 \boldsymbol{b}_2(x) + c_3 \boldsymbol{b}_3(x) = a_0 + a_1 x + a_2 x^2 = \boldsymbol{p}(x).$$

Identify powers of x

$$c_1 \boldsymbol{b}_1(x) + c_2 \boldsymbol{b}_2(x) + c_3 \boldsymbol{b}_3(x) = c_1 + c_2(2x^2+x+2) + c_3(-x^2+x) = (c_1 + 2c_2) \cdot 1 + (c_1 + c_2) \cdot x + (2c_2 - c_3) \cdot x^2$$

to obtain system $\boldsymbol{M}\boldsymbol{c} = \boldsymbol{a}$

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix},$$

Reduce matrix $\boldsymbol{M}$ to row-echelon form.

$$\boldsymbol{M} = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Since the row-echelon form of $\boldsymbol{M}$ does not have any zero rows, a unique solution of $\boldsymbol{M}\boldsymbol{c} = \boldsymbol{a}$ is found, and $\boldsymbol{B}$ is a basis for $\mathcal{P}_3$. Check in Maxima

```
(%i3) M: matrix([1, 2, 0],[1, 1, 0],[0, 2, -1]);
```

$$\text{(\%o3)} \quad \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & -1 \end{pmatrix}$$

```
(%i4) echelon(M);
```

$$\text{(\%o4)} \quad \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
(%i5)
```

Ex 2. $\mathcal{V} = \mathbb{R}^{2\times 2}$ the space of real-valued two-by-two matrices,

$$\boldsymbol{B} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \right)$$

Solution. Here are two approaches:

1. The basis can be reorganized as a set of $\mathbb{R}^4$ vectors

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \text{etc.}; \boldsymbol{B} = \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

   Reduce $\boldsymbol{B}$ to row echelon form

```
(%i45) B: matrix([1, 1, -2, 0],[0, 0, 1, 0],[0, 1, 1, 0],[0, 0, 1, 2]);
```

$$\text{(\%o45)} \quad \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

```
(%i47) echelon(B);
```

$$\text{(\%o47)} \quad \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
(%i48)
```

   Since the reduced row echelon form has no row of zeros $\boldsymbol{B}$ is a basis

2. Denote $\boldsymbol{B} = (\boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{B}_3, \boldsymbol{B}_4)$, and consider whether some general

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in \mathbb{R}^{2\times 2}$$

   can be expressed as a linear combination $\boldsymbol{A} = c_1\boldsymbol{B}_1 + c_2\boldsymbol{B}_2 + c_3\boldsymbol{B}_3 + c_4\boldsymbol{B}_4$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = c_1\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + c_2\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + c_3\begin{pmatrix} -2 & 1 \\ 1 & 1 \end{pmatrix} + c_4\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}.$$

   Indentify each component to obtain the system

$$\begin{cases} c_1 + c_2 - 2c_3 & = & a_{11} \\ c_2 + c_3 & = & a_{12} \\ c_3 & = & a_{21} \\ c_3 + 2c_4 & = & a_{22} \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{pmatrix}.$$

   Apart from a permutation of two rows, this is the same as matrix in solution procedure 1, and $\boldsymbol{B}$ does form a basis

Ex 3. $\mathcal{V}$ is the set of symmetric matrices, $\boldsymbol{A} \in \mathcal{V} \Rightarrow \boldsymbol{A}^T = \boldsymbol{A}$.

$$\boldsymbol{B} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right)$$

Solution. Since

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

cannot be expressed as a linear combination of elements in $\boldsymbol{B}$, the system is not a basis. This is verified if we form the matrix

$$\boldsymbol{B}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which has a row of zeros.

3. Find a basis for the subspace $\mathcal{S}$ of vector space $\mathcal{V}$. Specify the dimension of $\mathcal{S}$.

Ex 1.

$$\mathcal{S}=\left\{\left(\begin{array}{l} s+2t \\ -s+t \\ t \end{array}\right) | \, s,t\in\mathbb{R}\right\}, \mathcal{V}=\mathbb{R}^3$$

Solution. Recognize that the specification of $\mathcal{S}$ gives a linear combination with scalar coefficients s,t, and rewrite

$$\left(\begin{array}{l} s+2t \\ -s+t \\ t \end{array}\right)=s\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}+t\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

Construct

$$\boldsymbol{B}=(\ \boldsymbol{b}_1\ \ \boldsymbol{b}_2\ )=\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Check if columns of $\boldsymbol{B}$ are linearly independent,

$$s\boldsymbol{b}_1+t\boldsymbol{b}_2=\mathbf{0}\Rightarrow\left(\begin{array}{l} s+2t \\ -s+t \\ t \end{array}\right)=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

and since $s=t=0$ is the only solution, columns of $\boldsymbol{B}$ are linearly independent and span the subspace, hence are a basis for $\mathcal{S}$.

Ex 2.

$$\mathcal{S}=\left\{\begin{pmatrix} a & a+d \\ a+d & d \end{pmatrix} | \, a,d\in\mathbb{R}\right\}, \mathcal{V}=\mathbb{R}^{2\times 2}$$

Solution. Since

$$\begin{pmatrix} a & a+d \\ a+d & d \end{pmatrix}=a\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}+d\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\boldsymbol{B}=\left(\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\ \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\right)$$

is a basis.

Ex 3. $\mathcal{S}$ is the space of skew-symmetric matrices ($\boldsymbol{A}\in\mathcal{S}\Rightarrow\boldsymbol{A}^T=-\boldsymbol{A}$), $\mathcal{V}=\mathbb{R}^{2\times 2}$Solution. Since

$$\begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}=a\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\boldsymbol{B}=\left(\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)$$

is a basis.

Ex 4. $\mathcal{S}=\{p(x)|p(0)=0\}$, $\mathcal{V}=\mathcal{P}_2$.

Solution. Since $\boldsymbol{p}\in\mathcal{S}\Rightarrow\boldsymbol{p}=ax^2+bx$, $\boldsymbol{B}=\{x,x^2\}$ is a basis.

Ex 5. $\mathcal{S}=\{p(x)|p(0)=0,p(1)=0\}$, $\mathcal{V}=\mathcal{P}_3$.

Solution. Since $\boldsymbol{p}\in\mathcal{S}\Rightarrow\boldsymbol{p}=ax^3+bx^2+cx$, with $\boldsymbol{p}(1)=a+b+c=0\Rightarrow c=-(a+b)$, hence

$$\boldsymbol{p}=ax^3+bx^2-(a+b)x=a(x^3-x)+b(x^2-x),$$

such that $\boldsymbol{B}=\{x^3-x,x^2-x\}$ is a basis.

4. Find the coordinates of the vector $\boldsymbol{v}$ in the basis $\boldsymbol{B}$.

Ex 1.

$$\boldsymbol{B}=\begin{pmatrix}3&-2\\1&2\end{pmatrix},\boldsymbol{v}=\begin{pmatrix}8\\0\end{pmatrix},\mathcal{V}=\mathbb{R}^2$$

Solution. If $\boldsymbol{B}$ is a basis for $\mathcal{V}=\mathbb{R}^2$, then the vector $\boldsymbol{v}$ can be expressed as a linear combination of the basis vectors and that scalar coefficients are the coordinates of $\boldsymbol{v}$ in basis $\boldsymbol{B}$

$$\boldsymbol{v}=x_1\boldsymbol{b}_1+x_2\boldsymbol{b}_2=\boldsymbol{B}\begin{pmatrix}x_1\\x_2\end{pmatrix}\Rightarrow\text{Solve}\begin{pmatrix}3&-2\\1&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{pmatrix}8\\0\end{pmatrix}$$

to find the coordinates $x_1=2$, $x_2=-1$. Check in Maxima.

(%i5) `B:matrix([3,-2],[1,2]);`

(%o5) $\begin{pmatrix}3&-2\\1&2\end{pmatrix}$

(%i6) `v:matrix([8],[0]);`

(%o6) $\begin{pmatrix}8\\0\end{pmatrix}$

(%i7) `linsolve_by_lu(B,v);`

```
0 errors, 0 warnings
```

(%o7) $\left[\begin{pmatrix}2\\-1\end{pmatrix},\mathbf{false}\right]$

(%i8) `x: first(%);`

(%o8) $\begin{pmatrix}2\\-1\end{pmatrix}$

(%i9) `B.x`

(%o9) $\begin{pmatrix}8\\0\end{pmatrix}$

(%i10)

Ex 2.

$$\boldsymbol{B}=\begin{pmatrix}-2&-1\\4&1\end{pmatrix},\boldsymbol{v}=\begin{pmatrix}-2\\1\end{pmatrix},\mathcal{V}=\mathbb{R}^2$$

Solution. Solve by Gauss elimination (row reduction operations on the extended matrix)

$$\begin{pmatrix}-2&-1&-2\\4&1&1\end{pmatrix}\sim\begin{pmatrix}-2&-1&-2\\0&-1&-3\end{pmatrix}\sim\begin{pmatrix}-2&0&1\\0&1&3\end{pmatrix}\sim\begin{pmatrix}1&0&-\frac{1}{2}\\0&1&3\end{pmatrix}$$

and the coordinates are $\left(-\frac{1}{2},3\right)$. Check in Maxima

(%i48) `B:matrix([-2,-1],[4,1]);`

(%o48) $\begin{pmatrix} -2 & -1 \\ 4 & 1 \end{pmatrix}$

```
(%i49) v:matrix([-2],[1]);
```

(%o49) $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

```
(%i50) linsolve_by_lu(B,v);

0 errors, 0 warnings
```

(%o50) $\left[\begin{pmatrix} -\frac{1}{2} \\ 3 \end{pmatrix}, \mathbf{false}\right]$

Ex 3.

$$\boldsymbol{B} = \begin{pmatrix} 1 & 3 & 1 \\ -1 & -1 & 0 \\ 2 & 1 & 2 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \mathcal{V} = \mathbb{R}^3$$

Solution. Since $\boldsymbol{B} = (\ \boldsymbol{b}_1 \ \ \boldsymbol{b}_2 \ \ \boldsymbol{b}_3\ )$, and $\boldsymbol{v} = \boldsymbol{b}_3$ the coordinates are $(0,0,1)$.

Ex 4.

$$\boldsymbol{B} = \begin{pmatrix} 2 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \end{pmatrix}, \mathcal{V} = \mathbb{R}^3$$

Solution. With $\boldsymbol{B} = (\ \boldsymbol{b}_1 \ \ \boldsymbol{b}_2 \ \ \boldsymbol{b}_3\ )$, from

$$\boldsymbol{v} = c_1 \boldsymbol{b}_1 + c_2 \boldsymbol{b}_2 + c_3 \boldsymbol{b}_3$$

observe from second row that $c_1 = \frac{1}{2}$. Then

$$\frac{1}{2}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 + c_2 \\ 1 \\ \frac{1}{2} + 2c_2 + c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \end{pmatrix}$$

and the other coordinates are $c_2 = -1$, $c_3 = 2$. Verify in Maxima

```
(%i51) B:matrix([2,1,0],[2,0,0],[1,2,1]);
```

(%o51) $\begin{pmatrix} 2 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

```
(%i52) v:matrix([0],[1],[1/2]);
```

(%o52) $\begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \end{pmatrix}$

```
(%i53) linsolve_by_lu(B,v);
```

(%o53) $\left[\begin{pmatrix} \frac{1}{2} \\ -1 \\ 2 \end{pmatrix}, \mathbf{false}\right]$

```
(%i54)
```

Ex 5.

$$\boldsymbol{B} = \begin{pmatrix} 1 & x-1 & x^2 \end{pmatrix}, \boldsymbol{v} = -2x^2 + 2x + 3, \mathcal{V} = \mathcal{P}_2$$

Solution. With $\boldsymbol{B} = (\boldsymbol{b}_1 \;\; \boldsymbol{b}_2 \;\; \boldsymbol{b}_3)$ write

$$\boldsymbol{v} = -2\boldsymbol{b}_3 + 2\boldsymbol{b}_2 + 5\boldsymbol{b}_1 = -2(x^2) + 2(x-1) + 5(1) = -2x^2 + 2x - 2 + 5 = -2x^2 + 2x + 3,$$

and the coordinates are $(5, 2, -2)$.