

TEST 1

Solve the following problems (3 course points each). Present a brief motivation of your method of solution. Explicitly state any conditions that must be met for solution procedure to be valid. Organize your computation and writing so the solution you present is readily legible. No credit is awarded for statement of the final answer to a problem without presentation of solution procedure.

1. Verify for $c \in \mathbb{R}$ $y(x) = c^2 + cx + 2c + 1$ satisfies

$$y' = \frac{-x - 2 + \sqrt{x^2 + 4x + 4y}}{2} \quad (1)$$

on some open interval. Identify the open interval. Verify that $y_1 = -x(x + 4)/4$ also satisfies (1) on some open interval. Identify the interval.

2. Use variation of parameters and separation of variables to solve

$$xy' - 2y = \frac{x^6}{y + x^2}.$$

3. Find all (x_0, y_0) for which the initial value problem

$$y' = \frac{x^2 + y^2}{\ln(xy)}, y(x_0) = y_0$$

has a solution on some open interval that contains x_0 .

4. Find all functions N such that $(\ln(xy) + 2y \sin x) dx + N(x, y) dy = 0$ is exact.