Homework 03

Note: This will be the last homework in which model solutions are presented before due date. At this point, enough familiarity with TeXmacs and Mathematica has been obtained for students to complete subsequent homeworks.

1 Exercises

Exercise. RQ.1.22

Solution. $y' + 4xy = e^{-2x^2}$ is a linear ODE of form $y' + p(x)y = r(x)$, with $p(x) = 4x$, $r(x) = e^{-2x^2}$, with solution

$$y(x) = e^{-h(x)}\left(\int e^{h(x)}\, r(x)\, \mathrm{d}x + c\right), h(x) = \int p(x)\, \mathrm{d}x.$$

Compute:

$$h(x) = 2x^2$$

$$\int e^{h(x)}\, r(x)\, \mathrm{d}x = \int \mathrm{d}x = x,$$

hence

$$y(x) = e^{-2x^2}(x + c),$$

and $y(0) = -4.3$ gives $c = -4.3$. Verify:

```
In[1]:= ODE = y'[x]+4 x y[x] == Exp[-2x^2]
```

$y'(x) + 4\,x\,y(x) = e^{-2x^2}$

```
In[2]:= iCond = y[0]==-4.3
```

$y(0) = -4.3$

```
In[3]:= DSolve[{ODE,iCond},y[x],x]
```

$\{\{y(x) \to e^{-2x^2}\,(x - 4.3)\}\}$

```
In[4]:=
```

Exercise. PS1.1.6

Solution.

```
In[1]:=
```

Exercise. PS1.1.7

Solution.

```
In[1]:=
```

Exercise. PS1.1.8

Solution.

```
In[1]:=
```

Exercise. PS1.2.2

Solution.

```
In[1]:=
```

Exercise. PS1.2.3

Solution.

```
In[1]:=
```

Exercise. PS1.2.4

Solution.

```
In[1]:=
```

Exercise. PS1.3.5

SOLUTION.

```
In[1]:=
```

2 Problems

Problem. RQ.1.27

SOLUTION. The bacteria population $y(t)$ is given by $y' = ry$, $y(0) = y_0$, with solution $y(t) = e^{rt} y_0$. After 1 day, $y(1) = e^r y_0 = 1.25\, y_0 \Rightarrow r = \ln 1.25$. Population double condition is $e^{rt} = 2 \Rightarrow rt = \ln 2 \Rightarrow t = \ln 2 / \ln 1.25$. Population triple condition is $e^{rt} = 3 \Rightarrow rt = \ln 3 \Rightarrow t = \ln 3 / \ln 1.25$. Verify:

```
In[6]:= ODE = y'[t] == r y[t]
```

$y'(t) = r\, y(t)$

```
In[5]:= iCond = y[0] == y0
```

$y(0) = \mathrm{y0}$

```
In[9]:= sol[t_,r_] = y[t] /. DSolve[{ODE,iCond},y[t],t][[1,1]]
```

$\mathrm{y0} e^{rt}$

```
In[11]:= {sol[1,Log[1.25]]/y0,
          sol[Log[2]/Log[1.25],Log[1.25]]/y0,
          sol[Log[3]/Log[1.25],Log[1.25]]/y0}
```

$\{1.25, 2., 3.\}$

```
In[12]:=
```

Problem. PS1.1.17

SOLUTION.

```
In[1]:=
```

Problem. PS1.1.18

SOLUTION.

```
In[1]:=
```

Problem. PS1.3.22

SOLUTION.

```
In[1]:=
```

3 Projects

3.1 PS1.7.6

See Lesson 4, slide 3 for an example of Picard's iteration.