Homework 05

1 Exercises

Exercise. PS4.3.1

Solution. The general solution of the system $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{y}$ with

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

is $\boldsymbol{y}(t) = c_1 e^{\lambda_1 t}\boldsymbol{x}_1 + c_2 e^{\lambda_2 t}\boldsymbol{x}_2$, with λ_1, λ_2 eigenvalues of $\boldsymbol{A}$, and $\boldsymbol{x}_1, \boldsymbol{x}_2$ the corresponding eigenvectors, and $\boldsymbol{c} = \boldsymbol{X}^{-1}\boldsymbol{y}(0)$.

```
In[58]:= A={{1,1},{3,-1}}; MatrixForm[A]
```

$$\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

```
In[59]:= lambda = Eigenvalues[A]; Lambda = DiagonalMatrix[lambda]; MatrixForm[Lambda]
```

$$\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

```
In[62]:= X = Transpose[Eigenvectors[A]]; MatrixForm[X]
```

$$\begin{pmatrix} -1 & 1 \\ 3 & 1 \end{pmatrix}$$

The general solution is

$$\boldsymbol{y} = c_1 e^{-2t}\begin{pmatrix} -1 \\ 3 \end{pmatrix} + c_2 e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Here are some additional verifications not required in your homework solution, but given to show that the above result is confirmed by the Mathematica `DSolve` function.

```
In[74]:= y[t_]={y1[t],y2[t]};
         sol[t_]=Expand[DSolveValue[ {y'[t] == A . y[t], y1[0]==y10, y2[0]==y20},y[t],t]]
```

$$\left\{\frac{1}{2}e^{-4t}\text{y10}+\frac{1}{2}e^{-2t}\text{y10}-\frac{1}{2}e^{-4t}\text{y20}+\frac{1}{2}e^{-2t}\text{y20}, -\frac{1}{2}e^{-4t}\text{y10}+\frac{1}{2}e^{-2t}\text{y10}+\frac{1}{2}e^{-4t}\text{y20}+\frac{1}{2}e^{-2t}\text{y20}\right\}$$

```
In[49]:= sol[0]
```

$\{\text{y10}, \text{y20}\}$

```
In[75]:= c = Inverse[X]. {y10,y20};
         ans[t_]=Expand[c[[1]] Exp[lambda[[1]] t] X[[1]] + c[[2]] Exp[lambda[[2]] t]
         X[[2]]]
```

$$\left\{\frac{1}{2}e^{-4t}\text{y10}+\frac{1}{2}e^{-2t}\text{y10}-\frac{1}{2}e^{-4t}\text{y20}+\frac{1}{2}e^{-2t}\text{y20}, -\frac{1}{2}e^{-4t}\text{y10}+\frac{1}{2}e^{-2t}\text{y10}+\frac{1}{2}e^{-4t}\text{y20}+\frac{1}{2}e^{-2t}\text{y20}\right\}$$

```
In[78]:= Simplify[ans[t] == sol[t]]
```

True

Exercise. PS1.1.6

Solution.

```
In[1]:=
```

Exercise. PS1.1.7
SOLUTION.

```
In[1]:=
```

Exercise. PS1.1.8
SOLUTION.

```
In[1]:=
```

Exercise. PS1.2.2
SOLUTION.

```
In[1]:=
```

Exercise. PS1.2.3
SOLUTION.

```
In[1]:=
```

Exercise. PS1.2.4
SOLUTION.

```
In[1]:=
```

Exercise. PS1.3.5
SOLUTION.

```
In[1]:=
```

2 Problems

Problem. PS1.1.16
SOLUTION.

```
In[1]:=
```

Problem. PS1.1.17
SOLUTION.

```
In[1]:=
```

Problem. PS1.1.18
SOLUTION.

```
In[1]:=
```

Problem. PS1.3.22
SOLUTION.

```
In[1]:=
```

3 Projects

3.1 PS2.1.16