Lab01: An introduction to TeXmacs, Mathematica

1 TeXmacs

TeXmacs is an editor especially well suited for scientific work:

- Mathematical text is easily written, in legible form, e.g. $y' = f(x)$ is a differential equation and

$$\begin{cases} y' = f(x) \\ y(x_0) = y_0 \end{cases}, \tag{1}$$

  is an initial value problem.
- The text can be exported to LaTeX, HTML, PDF formats.
- Computations can be interspersed, such that one obtains a "living" document that contains the code used to produce results.
  - Mathematica solution of the ODE $y' = x + y$

```
In[1]:= ODE = y'[x]==x+y[x]
```

$y'(x) = y(x) + x$

```
In[2]:= DSolve[ODE,y[x],x]
```

$\{\{y(x) \to c_1 e^x - x - 1\}\}$

```
In[3]:=
```

2 Mathematica

Mathematica is a computational platform that combines symbolic, numerical and graphical calculations. Sample computations:

```
In[3]:= 10!
```

3628800

```
In[5]:= p4=Expand[(x+y)^4]
```

$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

```
In[6]:= Factor[p4]
```

$(x+y)^4$

```
In[8]:= Integrate[1/(1+x^2),x]
```

$\tan^{-1}(x)$

```
In[10]:= LaplaceTransform[Sin[t],t,s]
```

$\frac{1}{s^2+1}$

```
In[13]:= FourierTransform[Exp[-x^2],x,k]
```

$\frac{e^{-\frac{k^2}{4}}}{\sqrt{2}}$

```
In[14]:=
```

3 Euler's method

An analytical solution to an ODE may be impossible. This usually happens for non-linear, higher-order ODEs, e.g.,

```
In[7]:= ODE = y''[x] + x y[x] y'[x] == 0
```

$y''(x) + x\,y(x)\,y'(x) = 0$

```
In[8]:= DSolve[ODE,y[x],x]
```

$\text{DSolve}[y''(x) + x\,y(x)\,y'(x) = 0, y(x), x]$

Even if an analytical solution is obtainable, it might be too complicated to work with

```
In[24]:= f[x_,y_] = (x+y)^5;
         ODE = y'[x] == f[x,y[x]]
```

$y'(x) = (y(x) + x)^5$

```
In[25]:= DSolve[ODE,y[x],x]
```

$$\text{Solve}\left[\frac{1}{20}\left(4\log\left(y(x)+x+1\right)+(\sqrt{5}-1)\log\left((y(x)+x)^2+\frac{1}{2}(\sqrt{5}-1)\,(y(x)+x)+1\right)-(1+\sqrt{5})\log\left((y(x)+x)^2-\frac{1}{2}(1+\sqrt{5})\,(y(x)+x)+1\right)+2\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4\,y(x)+4\,x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)+2\sqrt{2\,(5+\sqrt{5})}\tan^{-1}\left(\frac{4\,y(x)+4\,x+\sqrt{5}-1}{\sqrt{2\,(5+\sqrt{5})}}\right)-20\,x\right)=c_1,y(x)\right]$$

In[26]:=

In such cases an approximation may be obtained numerically. Software systems like Mathematica contain sophisticated procedures that combine multiple algorithms to find a solution

```
In[42]:= IniCond = y[0]==0
```

$y(0)=0$

```
In[43]:= Y[x_] = y[x] /. NDSolve[{ODE,IniCond},y[x],{x,0,1}][[1,1]];
         YN = Table[{x,Y[x]},{x,0,1,0.2}]
```

0.	0.
0.2	0.000010759
0.4	0.00068596
0.6	0.00806314
0.8	0.0516308
1.	0.348729

In[44]:=

The simplest numerical algorithm to approximate an ODE is solution is Euler's method

$$Y_{i+1}=Y_i+h\,f(x_i,Y_i),$$

where $x_i=x_0+i\,h$, h is the step size, and $Y_i\cong y(x_i)$. Euler's method is applied repeatedly within a loop to obtain the approximation

```
In[29]:= h=0.2; x0=0.; y0=0.; xfinal=1.; nsteps=Floor[(xfinal-x0)/h]
```

5

```
In[45]:= YE = {y0,0,0,0,0,0};
         For[i=1, i<=nsteps, i++,
               YE[[i+1]] = YE[[i]] + h f[x0 + i h, YE[[i]]]
            ];
         {YN,YE}
```

{0., 0.}	{0.2, 0.000010759}	{0.4, 0.00068596}	{0.6, 0.00806314}	{0.8, 0.0516308}	{1., 0.348729}
0.	0.000064	0.00211364	0.0179415	0.0911634	0.400534

In[46]:=

Note that the approximate values from Euler's method are not the same as those from the Mathematica built-in NDSolve. In this lab we'll investigate why this happens.