Mini-Lab 04

In this mini-lab, we'll investigate the method of variation of constants for solving inhomogeneous ODEs of the form (§2.10)

$$y'' + p(x)\, y' + q(x)\, y = r(x). \tag{1}$$

One of the objectives of this mini-lab is to show how combined active-reading, hand-computation, structured writing, scratch notes, and symbolic computation come together to enable deeper understanding of topics within mathematical physics.

1 Method of variation of parameters

Seek solution of (1) as $y(x) = y_h(x) + y_p(x)$, with $y_h(x)$ the general solution of the homogeneous ODE

$$y'' + p(x)\, y' + q(x)\, y = 0, \tag{2}$$

and $y_p(x)$ a particular solution of the inhomogeneous ODE (1).

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In[22]:= rhs[y_,x_,p_,q_] := D[y,{x,2}] + p[x] D[y,x] + q[x] y;
         rhsODE = rhs[u[x] Subscript[y,1][x] + v[x] Subscript[y,2][x],x,p,q]
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$p(x)\,(y_1(x)\,u'(x) + u(x)\,y_1'(x) + y_2(x)\,v'(x) + v(x)\,y_2'(x)) + q(x)\,(u(x)\,y_1(x) + v(x)\,y_2(x)) + y_1(x)\,u''(x) + 2\,u'(x)\,y_1'(x) + u(x)\,y_1''(x) + y_2(x)\,v''(x) + 2\,v'(x)\,y_2'(x) + v(x)\,y_2''(x)$

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In[24]:= Simplify[rhsODE /. {Subscript[y,1]''[x] -> - p[x] Subscript[y,1]'[x] - q[x]
         Subscript[y,1][x],Subscript[y,2]''[x] -> - p[x] Subscript[y,2]'[x] - q[x]
         Subscript[y,2][x]}]
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$p(x)\,(y_1(x)\,u'(x) + y_2(x)\,v'(x)) + y_1(x)\,u''(x) + 2\,u'(x)\,y_1'(x) + y_2(x)\,v''(x) + 2\,v'(x)\,y_2'(x)$

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In[18]:= rhs[u[x] y1[x] + v[x] y2[x],x,p,q]
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$p(x)\,(u(x)\mathrm{y1}(x) + v(x)\mathrm{y2}(x))'(x) + q(x)\,(u(x)\mathrm{y1}(x) + v(x)\mathrm{y2}(x))(x) + (u(x)\mathrm{y1}(x) + v(x)\mathrm{y2}(x))''(x)$

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In[19]:=
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