Remark. Equations of the form

$$y'' + ay' + by = f(x) \tag{1}$$

with $a, b \in \mathbb{R}$, have wide-spread application throughout the sciences. Generally:

- the y'' term models *inertia*, the tendency of a system to maintain its current state
- the ay' term models *dissipation*, the transfer of energy to unresolved scales
- the by term models *storage*, the transfer of energy to potential form
- The f term models *external forcing*.

Remark. For $f = 0$, (1) is a subcase of $y'' + p(x)y' + q(x)y = 0$, hence the general solution is a linear combination $c_1y_1 + c_2y_2$ of two independent solutions y_1, y_2 of (1) with $c_1, c_2 \in \mathbb{R}$.

Remark. Guess a solution might be of form $y = e^{\lambda x}$, leading to the *characteristic equation*

$$\lambda^2 + a\lambda + b = 0,$$

with solutions

$$\lambda_{1,2} = \frac{1}{2}\left(-a \pm \sqrt{a^2 - 4b}\right),$$

and distinct cases:

1. Two real roots if $a^2 - 4b > 0$
2. Real double roots if $a^2 - 4b = 0$
3. Complex conjugate roots if $a^2 - 4b < 0$