The following model describes response of a system with inertia, dissipation, storage under excitation

$$my'' + cy' + ky = r(t),$$

especially with $r(t) = F\cos(\omega t)$ (periodic forcing).

```
In[6]:= rhs = m y''[t] + c y'[t] + k y[t]; r[t_] = F Cos[omega t];
        hODE = rhs == 0;
        hsol[t_,m_,c_,k_] = y[t] /. DSolve[hODE,y[t],t][[1,1]]
```

$$c_1\, e^{\frac{1}{2}t\left(-\frac{\sqrt{c^2-4km}}{m}-\frac{c}{m}\right)} + c_2\, e^{\frac{1}{2}t\left(\frac{\sqrt{c^2-4km}}{m}-\frac{c}{m}\right)}$$

```
In[2]:= psol[t_] = a Cos[omega t] + b Sin[omega t]
```

$$a\cos(\omega t) + b\sin(\omega t)$$

```
In[3]:= uCoef = Simplify[Evaluate[rhs /. y->psol] - r[t]]
```

$$\cos(\omega t)\,(a\,k - a\,m\,\omega^2 + b\,c\,\omega - F) + \sin(\omega t)\,(b\,(k - m\,\omega^2) - a\,c\,\omega)$$

```
In[4]:= sys = {Coefficient[uCoef,Cos[omega t],1],Coefficient[uCoef,Sin[omega t],1]}=={0,0}
```

$$\{a\,k - a\,m\,\omega^2 + b\,c\,\omega - F,\, b\,(k - m\,\omega^2) - a\,c\,\omega\} = \{0, 0\}$$

```
In[5]:= yp[t_,m_,c_,k_] = psol[t] /. Solve[sys,{a,b}][[1]]
```

$$\frac{c\,F\,\omega\sin(\omega t)}{c^2\,\omega^2 + k^2 - 2\,k\,m\,\omega^2 + m^2\,\omega^4} + \frac{F\,(k - m\,\omega^2)\cos(\omega t)}{c^2\,\omega^2 + k^2 - 2\,k\,m\,\omega^2 + m^2\,\omega^4}$$

```
In[6]:=
```

- $y(t) = y_h(t) + y_p(t)$

```
In[7]:= y[t_,m_,c_,k_] = hsol[t,m,c,k] + yp[t,m,c,k]
```

$$\frac{c\,F\,\omega\,\sin(\omega\,t)}{c^2\,\omega^2 + k^2 - 2\,k\,m\,\omega^2 + m^2\,\omega^4} + \frac{F\,(k - m\,\omega^2)\cos(\omega\,t)}{c^2\,\omega^2 + k^2 - 2\,k\,m\,\omega^2 + m^2\,\omega^4} + c_1\,e^{\frac{1}{2}t\left(-\frac{\sqrt{c^2-4\,k\,m}}{m}-\frac{c}{m}\right)} + c_2\,e^{\frac{1}{2}t\left(\frac{\sqrt{c^2-4\,k\,m}}{m}-\frac{c}{m}\right)}$$

```
In[8]:=
```

- Undamped forced oscillations

```
In[14]:= yNoDamp[t_] = Assuming[m>0, Simplify[y[t,m,0,m (Subscript[omega,0])^2] /. F->(f m) ]]
```

$$c_1\,e^{\sqrt{-\omega_0^2}(-t)} + c_2\,e^{\sqrt{-\omega_0^2}\,t} - \frac{f\cos(\omega\,t)}{\omega^2 - \omega_0^2}$$

```
In[15]:=
```

(See Lesson07.nb for examples)